03

In electrostatics, we have studied about the charges at rest. Here, we will study about charges in motion which constitute an electric current. Such electric currents occur naturally in many situations, e.g. in lightning, charges flow from the clouds to the earth through the atmosphere, which sometimes becomes disastrous, as the flow of charges is not steady. However, in our everyday life we see many devices such as a torch, a cell-driven clock, etc., where charges flow in steady manner.

CURRENT ELECTRICITY

|TOPIC 1|
Electric Current and Ohm's Law

CHAPTER CHECKLIST
- Electric Current and Ohm's Law
- Electrical Energy
- Cells, EMF and Internal Resistance
- Kirchhoff's Laws and its Applications

If we maintain the constant potential difference between two conductors, we get a constant flow of charge in a metallic wire connecting the two conductors. The flow of charge in metallic wire constitutes **electric current**. The branch of physics which deals with the charges in motion is called **current electricity**.

ELECTRIC CURRENT

It is defined as the rate of flow of electric charge through any cross-section of a conductor. It is denoted by I. Electric current can be expressed by

$$I = \frac{\text{Total charge flowing } (q)}{\text{Time taken } (t)}$$

If the charge dq flows through a conductor for small time dt, then $I = \frac{dq}{dt}$.

It means that the current through a conductor at a time is defined as the first derivative of charge passing through a cross-section of the conductor in a particular direction with respect to time.

If n = number of charges, e = electric charge and t = time,

then $$\boxed{I = \frac{q}{t} = \frac{ne}{t}}$$ [here, $q = ne$]

Conventionally, the direction of electric current is along the direction of motion of positive charges and opposite to the direction of motion of negative charges.

Electric current is not always steady and hence more generally, we can define the current as follows

Let Δq be the net charge flowing through a cross-section of the conductor in a particular direction during the time interval Δt [i.e. between times t and $(t+\Delta t)$] . Then, at time t, the current in the conductor is given by

$$dI(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$$

Current is a scalar quantity. Although it represents the direction of flow of positive charges, it does not follow the laws of vector addition (since the angle between the wires carrying current does not affect the total current). It follows the laws of scalar addition.

Unit of Electric Current

The SI unit of current is ampere and it is represented by A.

$$1 \text{ ampere (A)} = \frac{1 \text{ coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ coulomb per second}$$

or 1 Cs^{-1}. Current through a wire is said to be **one ampere,** if a charge of one coulomb flows through any cross-section of the wire in one second.

EXAMPLE |1| How many electrons pass through a lamp in 1 min, if the current is 300 mA? Given, the charge on an electron is 1.6×10^{-19} C.

Sol. Given, current, $I = 300 \text{ mA} = 300\times10^{-3}$ A

Charge on one electron, $e = 1.6\times10^{-19}$ C

Time, $t = 1$ min $= 60$ s

Charge passing through a lamp in 1 min, $q = I \times t$

$= 300\times10^{-3}\times60$

Let n electrons pass through the lamp in 1 min.

$$\therefore \quad q = ne \Rightarrow n = \frac{q}{e} = \frac{300\times10^{-3}\times60}{1.6\times10^{-19}}$$

$$= 1.125\times10^{20} \text{ C}$$

Important Points Related to Flow of Current

As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called conventional current. The direction of flow of electrons gives the direction of electronic current. Therefore , the direction of electronic current is opposite to that of conventional current.

If the current varies with time, it is represented by differential limit of q, i.e. $I = \frac{dq}{dt}$. Further, I is same, even when cross-sectional area is different at different points of the conductor.

Through a cross-section of the conductor in a time t, if a positive charge q_1 is flowing from A to B and a negative charge q_2 is flowing from B to A, then total current through the conductor is given by

$$I = \frac{q_1}{t} + \frac{q_2}{t} = \frac{q_1 + q_2}{t}$$

The electric current, which flows during the lightning, is of the order of tens of thousands of amperes. However, the current in our nerves is in microamperes.

Current Density

The current density at a point in a conductor is the ratio of the current at that point in the conductor to the area of cross-section of the conductor at that point.If a current I is distributed uniformly over the cross-section A of a conductor, then the current density at that point is

$$J = \frac{I}{A}$$

It is a characteristic property of a point inside the conductor. It is a vector quantity. Its direction at a point is the direction of flow of positive charge at that point.

The SI unit of current density is Am^{-2}.

EXAMPLE |2| An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wires carry an electric current of 10 A. Determine the current density in aluminium wire.

Sol. Given, diameter = 0.24 cm,

radius, $r = \frac{0.24\times10^{-2}}{2} = 0.12\times10^{-2}$ m

and current, $I = 10$ A

$\therefore$ Current density, $J = \frac{I}{A} = \frac{I}{\pi r^2}$

$$= \frac{10}{3.14\times(0.12\times10^{-2})^2} = 2.2\times10^6 \text{ Am}^{-2}$$

Electric Current in Conductors

All metals are good conductors of electricity. The electric conduction in them can be explained by an electron theory. In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it due to the strong attraction of the nuclear positive charge, but the electrons far from the nucleus experience a very weak attractive force.

Hence, the outer electrons can be removed easily from the atom (by rubbing or by heating the substance). In fact, a few outer electrons leave their atoms and move freely within the substance (in the vacant spaces between the atoms). These electrons called **free electrons** or **conduction electrons**. They carry the charge in the substance from one place to the other.Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it. In metals, this number is quite large ($\approx 10^{29}/m^3$).

Hence, metals are good conductors of electricity. Silver is the best conductor of electricity than copper, gold and aluminium, respectively.

In liquids and gases, electric conduction takes place by the movement of both positive and negative charge carribers unlike in metals, where the electric conduction occurs by the movement of negative charge carriers (electrons) only.

In case of a liquid conductor such as electrolytic solution, there are positive and negative charged ions, which can move on applying electric field, and hence generating the electric current. Whereas in case of a solid conductor (i.e. Cu, Fe, Ag, etc.) atoms are tightly bound to each other. They consist of large number of free electrons, which are responsible for the strong current in them when electric field is applied on them.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, i.e. no current on applying electric field. Such materials are called **insulators.** e.g. Wood, plastic, rubber, etc.

In our discussions, we will focus only on solid conductors in which the positive ions are at fixed positions and the current is carried by the negatively charged electrons.

Flow of Electric Charge

When no electric field is applied on a solid conductor, the free electrons in them move like molecules in a gas due to their thermal velocities. There is no preferential direction for the velocities of the electrons. The average thermal random velocity is zero. Due to this, there is no net flow of electric charge in a particular direction inside the conductor and hence no current flows in it.

When an electric field is applied on a solid conductor, in the shape of a cylinder of circular cross-section by attaching positively and negatively charged circular discs of a dielectric of the same radius as that of the solid conductor, at the two ends, an electric field is generated in the conductor from positive charged disc towards negative charged disc.

Electrolytic solutions are conductors of different types in which positive and negative charges can move.

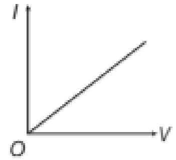
OHM'S LAW

It states that the current I flowing through a conductor is always directly proportional to the potential difference V across the ends of the conductor, provided that the physical conditions (temperature, mechanical strain, etc.) are kept constant. Mathematically,

$$I \propto V \quad \text{or} \quad V \propto I \quad \text{or} \quad \boxed{V = IR}$$

where, R is resistance of the conductor.

Its value depends upon the length, shape and the nature of the material of the conductor. It is independent of the values of V and I. Such conductors are said to obey Ohm's law. For an ohmic conductor, the graph between V and I is a straight line as shown in the figure.



EXAMPLE |3| A potential difference of 3 V is applied across a conductor through which 5 A of current is flowing. Determine the resistance of the conductor.

Sol. Given, potential difference, $V = 3\text{ V}$

Current, $I = 5\text{A}$

According to Ohm's law, $R = \frac{V}{I} \Rightarrow R = \frac{3}{5} = 0.6\,\Omega$

Resistance of a Conductor

It is defined as the ratio of the potential difference applied across the ends of the conductor to the current flowing through it.

Mathematically, $R = \frac{V}{I}$

The SI unit of resistance is ohm and denoted by Ω.

$$1\text{ ohm }(\Omega) = \frac{1\text{ volt (V)}}{1\text{ ampere (A)}} = 1\text{ volt/ampere (or V/A)}$$

The resistance of a conductor is said to be **one ohm**, if one ampere of current flows, when a potential difference of one volt is applied across the ends of the conductor.

Dimensional formula of electrical resistance is $[ML^2T^{-3}A^{-2}]$.

The resistance of the conductor depends upon the following factors

(i) It is directly proportional to the length of the conductor.

i.e. $$R \propto l \quad \ldots\text{(i)}$$

(ii) It is inversely proportional to the area of the cross-section of the conductor.

i.e. $$R \propto \frac{1}{A} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$R \propto \frac{l}{A} \quad \text{or} \quad \boxed{R = \rho\frac{l}{A}} \quad \ldots\text{(iii)}$$

where, ρ is the constant of proportionality known as **resistivity** or **specific resistance** of the conductor.

(iii) It depends upon the nature of the material and temperature of the conductor.

EXAMPLE |4| A negligible small current is passed through a wire length 15 m and uniform cross-section $6\times 10^{-7}\,m^2$ and its resistance is measured to be 5 Ω. What is the resistivity of the material at the temperature of the experiment? **NCERT**

Sol. Given, $R = 5\Omega$, $A = 6\times10^{-7}\,m^2$ and $l = 15m$

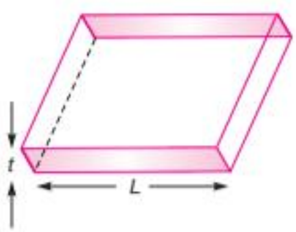
Let the resistivity of the material be ρ.

∴ Resistance of wire, $R = \rho \frac{l}{A}$

$$\Rightarrow \qquad \rho = \frac{RA}{l} = \frac{5\times 6\times 10^{-7}}{15} = 2\times10^{-7}\,\Omega\text{-m}$$

Thus, the resistivity of the material at the temperature of the experiment is $2\times10^{-7}\,\Omega\text{-m}$.

EXAMPLE |5| Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure. **(2010)**



Sol. Resistance between the shaded opposite faces is

$$R = \frac{\rho(L)}{A} = \frac{\rho L}{tL} = \frac{\rho}{t}$$

Note Here, R is independent of L.

Effect of Temperature on Resistance

Resistance of a metallic conductor at temperature t°C is given as

$$R_t = R_0\,(1+\alpha t + \beta t^2) \qquad \text{...(i)}$$

where, R_0 = resistance of conductor at 0°C and α, β are the temperature coefficients of resistance.

If the temperature t°C is not sufficiently large which is so in most of the practical cases, then Eq. (i) can be written as

$$R_t = R_0(1+\alpha t) \text{ or } \boxed{\alpha = \frac{R_t - R_0}{R_0 \times t}}$$

$$= \frac{\text{Change in resistance}}{\text{Original resistance} \times \text{Rise of temperature}}$$

Therefore, temperature coefficient of resistance is defined as the increase in resistance per unit original resistance per degree celsius or kelvin rise of temperature.

For metals, the value of α is positive, therefore, resistance of the metal increases with rise in temperature.

For insulators and semiconductors, the value of α is negative, therefore, resistance decreases with rise in temperature.

For alloys, the value of α is very small. The value of α is different at different temperatures. Temperature coefficient of resistance averaged over the temperature range t_1°C to t_2°C is given by

$$\boxed{\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}}$$

where, R_1 and R_2 are the resistances at t_1°C and t_2°C, respectively.

EXAMPLE |6| A silver wire has a resistance of 2.1 Ω at 27.5 °C and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver. **NCERT**

Sol. Given, $t_1 = 27.5$°C, $t_2 = 100$°C

$$R_1 = R_{27.5} = 2.1\,\Omega \text{ and } R_2 = R_{100} = 2.7\,\Omega$$

∴ Temperature coefficient of silver is given by

$$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)} = \frac{2.7 - 2.1}{2.1\,(100 - 27.5)} = 0.0039\,°C^{-1}$$

DRIFT OF ELECTRONS AND THE ORIGIN OF RESISTIVITY

Drift Velocity

It is defined as the average velocity with which the free electrons in a conductor get drifted towards the positive end of the conductor under the influence of an electric field applied across the conductor. It is denoted by v_d. The drift velocity of electron is of the order of $10^{-4}\,ms^{-1}$.

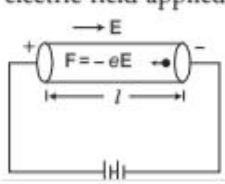
Let V be the potential difference applied across the ends of a conductor of length l, then the magnitude of electric field is

$$E = \frac{V}{l}$$

The direction of electric field is from positive to negative end of conductor as shown in figure. Since, the charge on an electron is $-e$ and each free electron in the conductor experiences a force F.

$$\therefore \qquad F = -eE$$

Here, negative sign indicates that the direction of force is opposite to that of electric field applied.



Current in a metallic conductor

Acceleration of each electron is given by

$$\mathbf{a} = -\frac{e\mathbf{E}}{m} \quad \left[\text{from Newton's second law, } a = \frac{F}{m}\right]$$

where, m is mass of the electron.

Under the effect of applied electric field, the free electrons accelerate and acquire a velocity component in a direction opposite to the direction of electric field in addition to their thermal velocities.

However, this gain in velocity of electron due to the electric field is very small and it is lost in the next collision with the ion/atom of the conductor.

At any instant of time, the velocity acquired by electron having thermal velocity $\mathbf{u}_1$ is given by

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{a}t_1$$

where, t_1 is the time elapsed as it has suffered its last collision with ion atom of conductor.

Similarly, $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{a}t_2, \ldots, \mathbf{v}_n = \mathbf{u}_n + \mathbf{a}t_n$

$\therefore$ Average velocity, $\mathbf{v}_d = \dfrac{\mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_n}{n}$

$$= \frac{(\mathbf{u}_1 + \mathbf{a}t_1) + (\mathbf{u}_2 + \mathbf{a}t_2) + \cdots + (\mathbf{u}_n + \mathbf{a}t_n)}{n}$$

$$= \frac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} + \frac{\mathbf{a}[t_1 + t_2 + \ldots + t_n]}{n}$$

We know that, $\dfrac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} = 0$

and $\dfrac{t_1 + t_2 + \ldots + t_n}{n}$ is called average time elapsed or **average relaxation time** and is denoted by τ. Its value is of the order of 10^{-14} s.

$$\therefore \quad \mathbf{v}_d = 0 + \mathbf{a}\tau \text{ or } \mathbf{v}_d = \mathbf{a}\tau \text{ or } \mathbf{v}_d = -\frac{e\mathbf{E}}{m}\tau$$

Negative sign shows that $\mathbf{v}_d$ is opposite to the direction of $\mathbf{E}$.

$$\therefore \quad \boxed{\text{Average drift velocity, } \mathbf{v}_d = \frac{e\mathbf{E}}{m}\tau}$$

Average relaxation time

= Mean free path of electron/drift speed of electron.

Relation between Drift Velocity and Electric Current

Consider a conductor of length l and A be the uniform area of cross-section.

$\therefore$ Volume of conductor $= Al$

If the conductor contains free electrons n per unit volume.

Then, number of free electrons in the conductor $= nAl$

If e is the charge of an electron, then total charge on all free electrons in the conductor is given by

$$q = nAle \quad \ldots(\text{i})$$

Time taken by the free electrons to cross the length of the conductor

$$t = \frac{l}{v_d} \quad \ldots(\text{ii})$$

Since, current is the rate of flow of the charge through conductor.

$$I = \frac{q}{t}$$

Using Eqs. (i) and (ii), we get

$$I = \frac{nAle}{l/v_d} \Rightarrow \boxed{I = neAv_d} \quad \ldots(\text{iii})$$

Eq. (iii) gives the relation between the current flowing through the conductor and drift velocity of the electron.

Putting the value of $v_d \left(= \dfrac{eE\tau}{m}\right)$ in Eq. (iii), we get

$$\boxed{I = \frac{Ane^2\tau E}{m}}$$

EXAMPLE |7| Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}\ \text{m}^2$ carrying a current of 1.5 A. Assume the density of conduction electrons to be $9 \times 10^{28}\ \text{m}^{-3}$.

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Sol. Given, cross-sectional area, $A = 1.0 \times 10^{-7}\ \text{m}^2$

Current, $I = 1.5\text{A}$

Electron density, $n = 9 \times 10^{28}\ \text{m}^{-3}$

Drift velocity, $v_d = ?$

We know that, $I = neAv_d$

$$\Rightarrow \quad v_d = \frac{I}{neA} = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.042 \times 10^{-3}\ \text{m/s}$$

Deduction of Ohm's Law

We know that, $I = \dfrac{Ane^2\tau E}{m}$ and $V_d = \dfrac{eE}{m}.\tau = \left(\dfrac{eV}{ml}\tau\right)$

$$\left[\text{as } E = \frac{V}{l}\right]$$

So, $$I = Anev_d = Ane\left(\frac{eV}{ml}\tau\right) = \left(\frac{Ane^2\tau}{ml}\right)V$$

or $$\frac{V}{I} = \frac{ml}{Ane^2\tau} = R = \text{constant}$$

$\therefore \qquad \frac{V}{I} = R \Rightarrow \boxed{V = RI}$

This is called Ohm's law.

Mobility

It is defined as the magnitude of drift velocity of charge per unit electric field applied. It is expressed as

$$\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)} = \frac{qE\tau / m}{E} = \frac{q\tau}{m}$$

where, τ is the **average relaxation time** of the charge while drifting towards the opposite electrode and m is the mass of the charged particle.

Mobility of electrons, $\mu_e = \frac{e\tau_e}{m_e}$.

and **mobility of holes,** $\mu_h = \frac{e\tau_h}{m_h}$.

where, τ_e and τ_h are average relaxation time for electrons and holes, respectively. m_e and m_h refer to mass of electrons and holes, respectively. Charge on either is e.

The SI unit of mobility is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$ and it is in the order of 10^4. The practical unit of mobility is $(Cm^2/V\text{-}s)$. Mobility is a positive quantity.

The total current in the conducting material is the sum of the currents due to the positive current carriers (holes) and negative current carriers (electrons).

EXAMPLE |8| Find the current flow through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that, electron mobility is $4.5 \times 10^{-6} m^2s^{-1}V^{-1}$ and charge on an electron is 1.6×10^{-19}C. The number density of electrons in copper wire is 8.5×10^{28} m^{-3}.

Sol. Given, length of copper wire, $l = 0.2$ m

Cross-sectional area, $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$

Potential difference, $V = 4$ V

Electron mobility, $\mu = 4.5 \times 10^{-6} \text{ m}^2\text{s}^{-1}\text{V}^{-1}$

Charge of an electron, $e = 1.6 \times 10^{-19}$ C

Number density of electrons, $n = 8.5 \times 10^{28} \text{ m}^{-3}$

We know that electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{4}{0.2} = 20 \text{ V/m}$$

$\therefore$ Current through the wire, $I = nAe\, v_d = nAe\mu E$

$[\because \mu = v_d / E]$

$= 8.5 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19} \times 4.5 \times 10^{-6} \times 20$

$= 1.22$ A

Limitations of Ohm's Law

The devices which do not obey Ohm's law are called **non-ohmic devices**, such as vacuum tubes, semiconductor diodes, transistors etc. The relation $\left(\frac{V}{I} = R\right)$ is valid for both, ohmic and non-ohmic devices.

For ohmic conductors, value of R is constant.

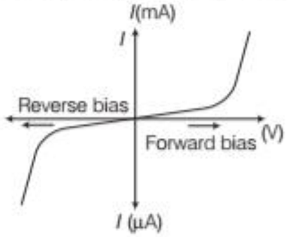
For non-ohmic devices, value of R is not constant, i.e. Ohm's law fails.

The limitations of Ohm's law are given below

Variation of potential difference with current

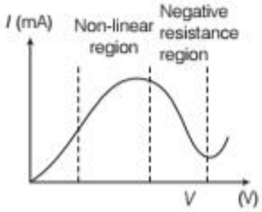
(i) Potential difference may vary non-linearly with current.

(ii) The variation of current with potential difference may depend upon sign of potential difference applied.



Variation of current according to the sign of potential difference

(iii) The relation between potential difference and current is not unique, i.e. there is more than one value of V for the same current I.



Graph of current *versus* potential difference for GaAs

RESISTIVITY OF VARIOUS MATERIALS

Specific resistance or resistivity of the material of a conductor is defined as the resistance of a unit length with unit area of cross-section of the material of the conductor, i.e. it is also defined as the resistance of unit cube of a material of the given conductor. The unit of resistivity is ohm-metre or Ω-m and its dimensional formula is $[ML^3T^{-3}A^{-2}]$.

Since, we know that $R = \rho \dfrac{l}{A} \Rightarrow \rho = \dfrac{RA}{l}$ ...(i)

Substituting the value of $R = \dfrac{ml}{ne^2A\tau}$ in Eq. (i),

We have, $$\rho = \left(\frac{ml}{ne^2A\tau}\right)\cdot\frac{A}{l}$$

∴ Resistivity of the material, $\boxed{\rho = \dfrac{m}{ne^2\tau}}$

From the above formula, it is clear that resistivity of a conductor depends upon the following factors:

(i) $\rho \propto \dfrac{1}{n}$, i.e. resistivity of a material is inversely proportional to the number density of free electrons (number of free electrons per unit volume).
As the free electron density depends upon the nature of material, so resistivity of a conductor depends on the nature of the material.

(ii) $\rho \propto 1/\tau$, i.e. resistivity of a material is inversely proportional to the average relaxation time τ of free electrons in the conductor.
As value of τ depends on the temperature of conductor, so resistivity of a conductor changes with temperature, as temperature increases, τ decreases, hence ρ increases.

Resistivity of Different Materials

Name of the materials	Resistivity at 0°C (Ω-m)	Name of the materials	Resistivity at 0°C (Ω-m)
1. **Conductors**		2. **Semiconductors**	
(i) **Metals**		Carbon	3.5×10^{-5}
Silver	1.6×10^{-8}	Germanium	0.46
Copper	1.7×10^{-8}	Silicon	2300
Aluminium	2.7×10^{-8}	3. **Insulators**	
Tungsten	5.6×10^{-8}	Glass	10^{10}-10^{14}
Iron	10×10^{-8}	Hard rubber	10^{13}-10^{16}
Platinum	11×10^{-8}	Mica	10^{11}-10^{15}
Mercury	98×10^{-8}	Wood	10^{8}-10^{11}
(ii) **Alloys**		Amber	5×10^{14}
Nichrome	-100×10^{-8}		
Manganin	48×10^{-8}		
Constantan	49×10^{-8}		

EXAMPLE |9| Find the time of relaxation between collision and free path of electrons in copper at room temperature.
(Given, resistivity of copper $=1.7\times10^{-8}$ Ω-m, density of electrons in copper $=8.5\times10^{28}\ m^{-3}$, charge on an electron $=1.6\times10^{-19}$ C, mass of electron $=9.1\times10^{-31}$ kg and drift velocity of free electrons $=1.6\times10^{-4}\ ms^{-1}$)

Sol. Given, $\rho = 1.7\times10^{-8}$ Ω-m, $n = 8.5\times10^{28}\ m^{-3}$,
$e = 1.6\times10^{-19}$ C, $m_e = 9.1\times10^{-31}$ kg
and $v_d = 1.6\times10^{-4}\ ms^{-1}$

We know that, $\rho = \dfrac{m_e}{ne^2\tau}$

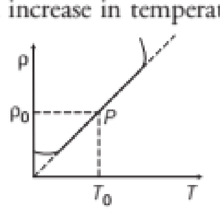
∴ Relaxation time,
$$\tau = \frac{m_e}{e^2n\rho} = \frac{9.1\times10^{-31}}{(1.6\times10^{-19})^2\times8.5\times10^{28}\times1.7\times10^{-8}}$$
$$= 2.5\times10^{-14}\ s$$

∴ Mean free path of electron (distance covered between two collisions)
$$= v_d\tau = 1.6\times10^{-4}\times2.5\times10^{-14} = 4\times10^{-18}\ m$$

Temperature Dependence of Resistivity

Resistivity of a metal conductor is given by
$$\rho = \rho_0[1+\alpha(T-T_0)] \quad ...(i)$$
where, ρ = resistivity at temperature T,
and ρ_0 = resistivity at temperature T_0
$$\Rightarrow \alpha = \frac{\rho-\rho_0}{\rho_0(T-T_0)} = \frac{d\rho}{\rho_0}\cdot\frac{1}{dT}$$

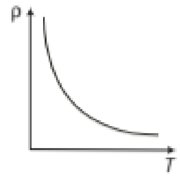
Thus, temperature coefficient of electrical resistivity is also defined as the fractional change in electrical resistivity $\dfrac{d\rho}{\rho_0}$ per unit change in temperature dT.
For metals, the value of α is **positive**, therefore resistivity of metal increases with increase in temperature.



Resistivity as a function of temperature for metals

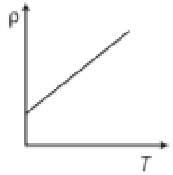
Eq.(i) implies that a graph of ρ plotted against T would be a straight line. At temperatures much lower than 0°C, the graph deviates considerably from a straight line. Eq.(i) can be used approximately over a limited range of T around any reference temperature T_0, where the graph can be approximated as a straight line.

For semiconductors, the resistivity decreases with increase in temperature.



Resistivity as a function of temperature for semiconductors

For alloys, the resistivity is very large, but has weak dependence on temperature.



Resistivity as a function of temperature for alloys

Electric fuse is made of an alloy of zinc, copper, silver and aluminium. This is because alloys have low resistivity. This causes the wire to melt, if a current more than safe current flows through the circuit.

CONDUCTANCE AND CONDUCTIVITY

Conductance

It is defined as the reciprocal of resistance of a conductor. It is expressed as

$$\boxed{G = \frac{1}{R}}$$

Its SI unit is mho (Ω^{-1}) or siemen (S).

The dimensional formula of conductance is $[M^{-1}L^{-2}T^{3}A^{2}]$.

Conductivity

It is defined as the reciprocal of resistivity of a conductor. It is expressed as

$$\boxed{\sigma = \frac{1}{\rho}}$$

The SI unit is mho per metre $(\Omega^{-1}\,m^{-1})$ or siemen per metre (S/m).

The dimensional formula of conductivity is $[M^{-1}L^{-3}T^{3}A^{2}]$.

EXAMPLE |10| A wire carries a current of 0.5 A, when a potential difference of 1.5 V is applied across it. What is its conductance? If the wire is of length 3m and area of cross-section 5.4 mm^2, then calculate its conductivity.

Sol. Here, $I = 0.5$ A, $V = 1.5$ V, $l = 3$ m,

$$A = 5.4\,\text{mm}^2 = 5.4 \times 10^{-6}\,\text{m}^2$$

$\therefore$ New resistance, $R = \frac{V}{I} = \frac{1.5}{0.5} = 3\,\Omega$

$\therefore$ Conductance, $G = \frac{1}{R} = \frac{1}{3} = 0.33$ S

and electrical conductivity,

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{3 \times 5.4 \times 10^{-6}} = 1.85 \times 10^{5}\,\text{Sm}^{-1}$$

Relation between J, σ and E (Microscopic form of Ohm's Law)

Since, the relation between the current flowing through the conductor and drift velocity of electron is given by

$$I = nAe\,v_d$$

$$\therefore \quad I = nAe\left(\frac{eE}{m}\tau\right) = \frac{nAe^2\tau E}{m}$$

$$\Rightarrow \quad \frac{I}{A} = \frac{ne^2\tau E}{m} \Rightarrow J = \frac{ne^2\tau E}{m} \qquad \left[\because J = \frac{I}{A}\right]$$

$$\text{or} \quad J = \frac{1}{\rho}E \qquad \left[\because \rho = \frac{m}{ne^2\tau}\right]$$

$$\therefore \quad \boxed{J = \sigma E} \qquad \left[\because \sigma = \frac{1}{\rho}\right]$$

It is a **microscopic** form of **Ohm's law**.

Classification of Materials in Terms of Conductivity

On the basis of conductivity, the materials can be classified into the following categories

Insulators

These are materials whose electrical conductivity is either very small or nil, e.g. glass, rubber, etc.

Conductors

These are materials whose electrical conductivity is very high, e.g. silver, aluminium, etc.

Semiconductors

These are those materials whose electrical conductivity lies in between that of insulators and conductors, e.g. germanium, silicon, etc.

Note **Thermistor** A thermistor is a heat sensitive device whose resistivity changes very rapidly with change of temperature.

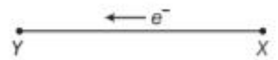
A thermistor can have a resistance in the range of $0.1\,\Omega$ to $10^7\,\Omega$, depending upon its composition.

Superconductivity The resistivity of certain metal or alloy drops to zero, when they are cooled below a certain temperature is called superconductivity. It was observed by Prof. Kamerlingh in 1911.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Twenty million electrons reaches from point X to point Y in two micro second as shown in the figure. Direction and magnitude of the current is



(a) 1.5×10^{-10} A from X to Y
(b) 1.6×10^{-6} A from Y to X
(c) 1.5×10^{-13} A from Y to X
(d) 1.6×10^{-4} A from X to Y

2. The relation between electric current density (J) and drift velocity (v_d) is

(a) $J = nev_d$ (b) $J = \frac{ne}{v_d}$
(c) $J = \frac{v_d e}{n}$ (d) $J = nev_d^2$

where, e is the charge of electron and n is the number of electrons.

3. If drift velocity of electron is v_d and intensity of electric field is E, then which of the following relation obeys the Ohm's law?

(a) v_d = constant (b) $v_d \propto E$
(c) $v_d = \sqrt{E}$ (d) $v_d \propto E^2$

4. Which of the following characteristics of electrons determines the current in a conductor? **NCERT Exemplar**

(a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity

5. The dimensional formula of resistance is

(a) $[ML^2T^{-2}A^{-2}]$ (b) $[M^2L^3T^{-2}A^{-2}]$
(c) $[ML^2T^{-3}A^{-2}]$ (d) $[ML^3T^{-3}A^{-3}]$

6. The resistance of a 10 m long wire is 10 Ω. Its length is increased by 25% by stretching the wire uniformly.
The resistance of wire will change to

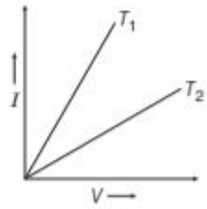
(a) 12.5 Ω (b) 14.5 Ω
(c) 15.6 Ω (d) 16.6 Ω

7. Multiplication of resistivity and conductivity of any conductor depends on

(a) cross-section (b) temperature
(c) length (d) None of these

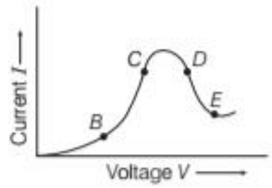
VERY SHORT ANSWER Type Questions

8. I-V graph for a metallic wire at two different temperatures T_1 and T_2 is as shown in the figure below.



Which of the two temperatures is lower and why? **Delhi 2015**

9. Graph showing the variation of current *versus* voltage for a material GaAs is shown in the figure. Identify the region of



(i) negative resistance.
(ii) where Ohm's law is obeyed. **All India 2015**

10. Why are alloys used for making standard resistance coils? **NCERT Exemplar**

11. Two materials Si and Cu, are cooled from 300 K to 60 K. What will be the effect on their resistivity? **Foreign 2013**

12. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction? **Delhi 2012**

13. Define the term mobility of charge carriers in a conductor. Write its SI unit. **Delhi 2014**

14. The relaxation time τ is nearly independent of applied E field, whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law, whereas the second fact leads to variation of ρ with temperature. Elaborate why? **NCERT Exemplar**

15. Is the motion of a charge across junction momentum conserving? Why or why not? **NCERT Exemplar**

16. Specific resistances of copper, silver and constantan are 1.78×10^{-6} Ω-cm, 10^{-6} Ω-cm and 48×10^{-6} Ω-cm, respectively. Which is the best conductor and why?

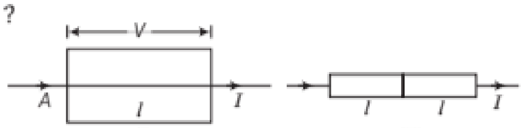
17. For wiring in the home, one uses Cu wires or Al wires. What considerations are involved in this? **NCERT Exemplar**

SHORT ANSWER Type Questions

18. What conclusion can you draw from the following observations on a resistor made of alloy manganin? **NCERT**

Current (in A)	Voltage (in V)	Current (in A)	Voltage (in V)
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

19. A metal rod of square cross-sectional area A having length l has current I flowing through it when a potential difference of V volt is applied across its ends (Fig. I). Now, the rod is cut parallel to its length into two identical pieces and joined as shown in Fig. II. What potential difference must be maintained across the length of $2l$, so that the current in the rod is still I?



(I) (II)

Foreign 2016

20. A conductor of length l is connected to a DC source of potential V. If the length of the conductor is tripled by gradually stretching it, keeping V constant, how will
(i) drift speed of electrons and
(ii) resistance of the conductor be affected? Justify your answer. **Foreign 2012**

21. Using the concept of drift velocity of charge carriers in a conductor, deduce the relationship between current density and resistivity of the conductor. **Delhi 2015 C**

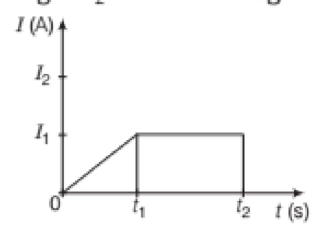
22. (i) A wire of resistivity ρ is stretched to three times its length. What will be its new resistivity?
(ii) In what manner, do the relaxation time in the good conductor change when its temperature increases?

23. Define mobility of a charge carrier. Write the relation expressing mobility in terms of relaxation time. Give its SI unit. **All India 2013**

24. Draw a plot showing the variation of resistivity of a (i) conductor and (ii) semiconductor, with the increase in temperature. How does one explain this behaviour in terms of number density of charge carriers and the relaxation time? **Delhi 2014 C**

LONG ANSWER Type I Questions

25. (i) Deduce the relation between current I flowing through a conductor and drift velocity $\mathbf{v}_d$ of the electrons.
(ii) Figure shows a plot of current I flowing through the cross-section of a wire *versus* the time t. Use the plot to find the charge flowing in t_2 second through the wire.



26. Define relaxation time of the free electrons drifting in a conductor. How it is related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material. **All India 2012**

27. Find the relation between drift velocity and relaxation time of charge carriers in a conductor.
A conductor of length L is connected to a DC source of emf E. If the length of the conductor is tripled by stretching it, keeping E constant, explain how its drift velocity would be affected. **Delhi 2015**

28. (i) Define the term of drift velocity.
(ii) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend?

(iii) Why alloys like constantan and manganin are used for making standard resistors? **Delhi 2016**

29. Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behaviour explained? **Delhi 2011**

30. A conductor of length l is connected to a DC source of potential V. If the length of the conductor is tripled by gradually stretching it, keeping V constant, how will

(i) drift speed of electrons and

(ii) resistance of the conductor be affected? Justify your answer. **Foreign 2012**

31. (a) Define the term 'conductivity' of a metallic wire. Write its SI unit.

(b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence, obtain the relation between current density and the applied electric field E. **CBSE 2018**

LONG ANSWER Type II Question

32. (i) Derive an expression for drift velocity of electrons in a conductor. Hence, deduce Ohm's law.

(ii) A wire whose cross-sectional area is increasing linearly from its one end to the others, is connected across a battery of V volts. Which of the following quantities remain constant in the wire?

(a) Drift speed (b) Current density

(c) Electric current (d) Electric field

Justify your answer. **Delhi 2017**

NUMERICAL PROBLEMS

33. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance R_A to R_B. **NCERT Exemplar**

34. A wire is stretched to increase its length by 5%. Calculate percentage change in its resistance.

35. At room temperature (27°C), the resistance of a heating element is 100 Ω. What is the temperature of the element, if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4}\,°C^{-1}$. **NCERT**

36. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element, if the room temperature is 27°C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}\,°C^{-1}$. **NCERT**

37. A resistance coil marked 3Ω is found to have a true resistance of 3.115Ω at 300 K. Calculate the temperature at which marking is correct. Temperature coefficient of resistance of the material of coil is $4.2 \times 10^{-3}\,°C^{-1}$. **All India 2014**

HINTS AND SOLUTIONS

1. (b) Given, number of electrons,

$$N = 20000000 = 2 \times 10^7$$

Total charge on twenty million electrons is

$$q = Ne$$
$$= 2 \times 10^7 \times 1.6 \times 10^{-19}\,C \qquad (\because e = 1.6 \times 10^{-19}\,C)$$
$$= 3.2 \times 10^{-12}\,C$$

Now time taken by twenty million electrons to pass from point X to point Y is $t = 2\mu s = 2 \times 10^{-6}\,s$

$$I = \frac{q}{t} = \frac{3.2 \times 10^{-12}}{2 \times 10^{-6}} = 1.6 \times 10^{-6}\ A$$

Since, the direction of the current is always opposite to the direction of flow of electrons. Therefore due to flow of electrons from point X to point Y, the current will flow from point Y to point X.

2. (a) Current density, $J = \frac{I}{A}$, $I = ne\,A\,v_d \Rightarrow J = nev_d$

3. (b) Drift velocity $v_d = -\frac{eE}{m}\tau \Rightarrow v_d \propto E$

4. (a) The relationship between current and drift speed is given by

$$I = ne\,Av_d$$

Here, I is the current and v_d is the drift velocity.

So, $I \propto v_d$

Thus, only drift velocity determines the current in a conductor.

5. (c) Resistance, $R = \rho\frac{l}{A}$

$$= \frac{[ML^3T^{-3}A^{-2}][L]}{[L^2]} = [ML^2T^{-3}A^{-2}]$$

6. (c) Given, $l_1 = l + \frac{25}{100} l = \frac{5l}{4}$.

Since, volume of wire remains unchanged on increasing length, hence

$$\Rightarrow \quad \begin{pmatrix} A_1 l_1 = Al \\ A_1 \times \frac{5l}{4} = Al \end{pmatrix} \quad \text{or} \quad A_1 = 4A/5$$

Given, $R = \rho l/A = 10\Omega$ and $R_1 = \frac{\rho l_1}{A_1} = \frac{\rho 5l/4}{4A/5} = \frac{25\rho l}{16A}$

$$\therefore \quad R_1 = \frac{25}{16} \times 10 = \frac{250}{16} = 15.6\,\Omega$$

7. (d) Resistivity and conductivity of conductor depends on the nature of substance.

8. Since, slope of 1> slope of 2

$\therefore \quad R_1 < R_2$

Also, we know that resistance is directly proportional to the temperature.

Therefore, $T_2 > T_1$.

9. (i) DE is the region, of negative resistance because the slope of curve in this part is negative.

(ii) BC is the region, where Ohm's law is obeyed because in this part, the current varies linearly with the voltage.

10. Alloys have small value of temperature coefficient of resistance with less temperature sensitivity. This keeps the resistance of wire almost constant even in small temperature change. Thus, alloy also has high resistivity for given length and cross-sectional area of conductor.

11. In silicon, the resistivity increases with decrease in temperature.

In copper, the resistivity decreases with decrease in temperature.

12. Yes, all the free electrons drift in the same direction.

13. Mobility of charge carriers inside conductor is defined as the magnitude of drift velocity of charge per unit electric field applied.

SI unit of mobility is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$.

14. Relaxation time is inversely proportional to the velocities of electrons and ions. The applied electric field produces the insignificant change in velocities of electrons at the order of 1 mm/s, whereas the change in temperature T affects velocities at the order of 10^2 m/s.

This decreases the relaxation time considerably in metals and consequently resistivity of metal or conductor increases as, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

15. When an electron approaches a junction, in addition to the uniform electric field **E** facing it normally, it keeps the drift velocity fixed, as drift velocity depends on E by the relation of drift velocity, $v_d = \frac{eE\tau}{m}$.

This results into accumulation of charges on the surface of wires at the junction. These produce an additional electric field. These fields change the direction conserving momentum. Thus, the motion of a charge junction is not momentum conserving.

16. The best conductor is silver because electrical conductivity is inversely proportional to the resistivity and resistivity of silver is least.

17. The Cu wires or Al wires are used for wiring in the home.The main considerations involved in this process are cost of metal and good conductivity of metal.

18. Here, Ohm's law is valid because ratio of voltage and current for different readings is same.

Also, the resistivity of alloy manganin is nearly independent of temperature.

19. From Ohm's law, we have $V = IR$

$$\Rightarrow \quad V = I\rho\frac{l}{A} \quad \left[\because R = \rho\frac{l}{A}\right] \quad \text{...(i)}$$

When the rod is cut parallel and rejoined by length, the length of the conductor becomes $2l$, whereas the area decrease to $\frac{A}{2}$. If the current remains the same, then the potential changes as

$$V = I\rho\frac{2l}{A/2} = 4 \times I\rho\frac{l}{A} = 4V \quad \text{[using Eq. (i)]}$$

The new potential applied across the metal rod will be four times the original potential (V).

20. The potential, V = constant, $l' = 3l$

(i) Drift speed of electrons, $v_d = \frac{V}{nel\rho}$

$$v_d \propto \frac{1}{l} \quad [\because \text{other factors are constant}]$$

So, when length is tripled, drift velocity gets one-third.

(ii) Resistance of conductor is $R = \rho\frac{l}{A}$.

Here, wire is stretched to triple its length, that means the mass of the wire remains same in both conditions.

Before stretching mass = After stretching mass

$$M_1 = M_2$$

$$\Rightarrow \quad V_1\rho_1 = V_2\rho_2 \quad [\because \rho_1 = \rho_2]$$

$$\text{or} \quad A_1 l_1 = A_2 l_2$$

Since, length is tripled after stretching.

$$\therefore \quad A_1 l = A_2(3l) \quad \text{or} \quad A_2 = \frac{A_1}{3}$$

Hence, $R' = \rho\frac{l'}{A'} = \rho\frac{3l}{A/3} = \frac{9\rho l}{A} \Rightarrow R' = 9R$

Thus, new resistance is 9 times of its original value.

21. Refer to text on page 130.

22. (i) Resistivity is a property of the material, it does not depend on the dimensions of the wire. Thus, when the wire is stretched, then its resistivity remains same.

(ii) Refer to text on page 129.

23. Refer to text on page 128.

24. Refer to text on pages 128 and 129.

25. (i) Refer to text on page 127.

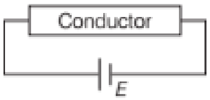
(ii) Area under I-t curve on t-axis is charge flowing through the conductor.

$$Q = \frac{1}{2} \times t_1 \times I_1 + (t_2 - t_1) \times I_1$$

26. Refer to text on pages 127 and 128.

27. Refer to text on page 127.

Source of emf E is shown in the figure below



Suppose initial length of the conductor, $l_2 = l_0$.

New length, $l_f = 3l_0$

We know that,

drift velocity, $v_d \propto E_0$ [where, E_0 = electric field]

Thus, $\frac{(v_d)_f}{(v_d)_i} = \frac{(E_0)_f}{(E_0)_i}$

$$= \frac{E / l_f}{E / l_i} = \frac{l_i}{l_f} = \frac{l_0}{3l_0} = \frac{1}{3}$$

$$\Rightarrow \quad (v_d)_f = \frac{(v_d)_i}{3}$$

Thus, drift velocity decreases three times.

28. (i) Refer to text on page 126.

(ii) Refer to text on pages 127, 128 and 129.

(iii) Alloys like constantan and manganin are used for making standard resistor because the resistivity of these alloys here weak dependent on the temperature.

29. Refer to text on pages 129 and 130.

30. When a wire is stretched, then there is no change in the matter of the wire, hence its volume remains constant.

The potential V = constant, $l' = 3l$

(i) Drift speed of electrons $= \frac{V}{nel\rho}$

$\therefore \quad v \propto \frac{1}{l}$ [$\because$ other factors are constants]

So, when length is tripled, drift velocity gets one-third.

$$V_1 = V_2$$

$$\therefore \quad A_1 l_1 = A_2 l_2$$

$$A_1 l = A_2 (3l)$$

$\Rightarrow$ [$\because$ length is tripled after stretching]

$$\therefore \quad A_2 = \frac{A_1}{3}$$

i.e. When length is tripled area of cross-section is reduced to $\frac{A}{3}$.

Hence, $R = \rho \frac{l'}{A'} = \rho \frac{3l}{\frac{A}{3}}$

$$= 9\rho \frac{l}{A} = 9R$$

Thus, new resistance will be 9 times of its original value.

31. (a) **Conductivity** The reciprocal of resistivity of a conductor is known as conductivity. It is expressed as

$$\sigma = \frac{1}{\rho}$$

The SI unit of conductivity is mho per metre ($\Omega^{-1} m^{-1}$).

(b) We know that, drift velocity is given by

$$v_d = \frac{eE\tau}{m} \quad \text{... (i)}$$

where, e = electric charge,

E = applied electric field,

τ = relaxation time and m = mass of electron.

But $E = \frac{V}{l}$ (i.e. potential gradient)

$$\therefore \quad v_d = \left(\frac{e\tau}{m}\right)\left(\frac{V}{l}\right) \quad \text{... (ii)}$$

From the relation between current and drift velocity,

$$I = neAv_d \quad \text{...(iii)}$$

(where, n = number of density of electrons)

Putting the value of Eq. (ii) in Eq. (iii), we get

$$I = neA\left(\frac{e\tau V}{ml}\right) \text{ or } I = \left(\frac{ne^2 A\tau}{ml}\right) V$$

$$\text{or} \quad V = \left(\frac{ml}{ne^2 A\tau}\right) I \quad \text{... (iv)}$$

But according to Ohm's law, $V = IR$ ... (v)

From Eqs. (iv) and (v), we get

$$R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A} \quad \text{... (vi)}$$

$$\text{Also,} \quad R = \rho \frac{l}{A} \quad \text{... (vii)}$$

From Eqs. (vi) and (vii), we get

$\rho = \frac{m}{ne^2\tau}$ = resistivity of conductor.

As reciprocal of resistivity of conductor is known as conductivity.

$\therefore$ Conductivity, $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

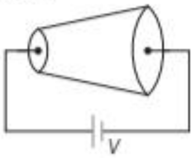
Now, we know that, current density, $J = \frac{I}{A}$

$$\text{or} \quad J = \frac{neAv_d}{A} = nev_d = \left(\frac{ne^2\tau}{m}\right)E \quad \left(\because v_d = \frac{eE\tau}{m}\right)$$

$$\therefore \quad J = \sigma E \quad \left(\because \sigma = \frac{ne^2\tau}{m}\right)$$

32. (i) Refer to text on pages 126, 127 and 128.

(ii) The setup is shown in the figure. Here, electric current remains constant throughout the length of the wire. Electric field also remains constant which is equal to $\frac{V}{l}$.



Current density and hence drift speed changes.

33. The resistance of first conductor, $R_A = \frac{\rho l}{\pi (0.5 \times 10^{-3})^2}$

The resistance of second conductor,

$$R_B = \frac{\rho l}{\pi[(10^{-3})^2 - (0.5 \times 10^{-3})^2]}$$

Now, the ratio of two resistors is given by

$$\frac{R_A}{R_B} = \frac{(10^{-3})^2 - (0.5 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} = 3:1$$

34. When a wire is stretched, its volume remains constant, hence

$$l_1 A_1 = l_2 A_2 = V \quad \text{[where, } V = \text{volume]}$$

Now, $R_1 = \frac{\rho l_1}{A} = \frac{\rho l_1 \times l_1}{l_1 A_1} = \frac{\rho l_1^2}{V}$, i.e. $R_1 \propto l_1^2$

Hence, $\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{\left(l_1 + \frac{5}{100} l_1\right)^2}{l_1^2} = 1.1025$

$$\frac{R_2}{R_1} = 1.1025 \quad \text{...(i)}$$

$\therefore$ % Change in resistance $= \frac{R_2 - R_1}{R_1} \times 100$

$= \left(\frac{R_2}{R_1} - 1\right) \times 100 = (1.1025 - 1) \times 100$ [from Eq. (i)

$= 10.25\%$

35. Given, resistance of heating element at temperature $27°C$, $R_{27} = 100\,\Omega$

Resistance of heating element at temperature $t°C$,

$$R_t = 117\,\Omega$$
$$\alpha = 1.70 \times 10^{-4} °C^{-1}, t = ?$$

By using the formula of temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$ ...(i)

Here, $R_2 = R_t$, $R_1 = R_{27}$, $t_2 = t$ and $t_1 = 27°C$

Such that, $\alpha = \frac{R_t - R_{27}}{R_{27}(t - 27)}$

Substituting given values in Eq. (i), we get

$$1.70 \times 10^{-4} = \frac{117 - 100}{100\,(t - 27)} \text{ or } t - 27 = \frac{17}{100 \times 1.70 \times 10^{-4}}$$

or $t = 1000 + 27 = 1027\,°C$

36. Given, potential difference = 230 V

Initial current at 27 °C = $I_{27°C} = 3.2$ A

Final current at t°C = $I_{t°C} = 2.8$ A

Room temperature = 27 °C

Temperature coefficient of resistance, $\alpha = 1.70 \times 10^{-4}\,°C^{-1}$

Resistance at 27 °C, $R_{27°C} = \frac{V}{I_{27°C}} = \frac{230}{3.2} = \frac{2300}{32}\,\Omega$

Resistance at t°C, $R_{t°C} = \frac{V}{I_{t°C}} = \frac{230}{2.8} = \frac{2300}{28}\,\Omega$

Temperature coefficient of resistance

$$\alpha = \frac{R_t - R_{27}}{R_{27}(t - 27)} \Rightarrow 1.70 \times 10^{-4} = \frac{\frac{2300}{28} - \frac{2300}{32}}{\frac{2300}{32}(t - 27)}$$

or $t - 27 = \frac{82.143 - 71.875}{71.875 \times 1.70 \times 10^{-4}} = 840.347$

or $t = 840.3 + 27 = 867.3\,°C$

Thus, the steady temperature of heating element is 867.3 °C.

37. 290.2K, refer to Sol. of Q. 35.

|TOPIC 2|

Electrical Energy

ELECTRICAL ENERGY AND POWER

Electrical Energy

It is defined as the total work done W by the source of emf V in maintaining the electric current I in the given circuit for a specified time t.

According to Ohm's law, we have

$$V = IR$$

Total charge that crosses the resistor is given by $q = It$

Energy gained is given by

$$E = W = Vq = V(It) = VIt$$

$$= [IR]It = I^2Rt \qquad [\because V = IR]$$

$$= \left[\frac{V}{R}\right]^2 Rt = \frac{V^2t}{R} \qquad \left[\because I = \frac{V}{R}\right]$$

$$\therefore \boxed{E = VIt = I^2Rt = \frac{V^2}{R}t}$$

The SI unit of electrical energy is joule (J),

where, 1 joule = 1 volt × 1 ampere × 1 sec = 1 watt × 1 sec

Commercial Unit of Electrical Energy

To measure the electrical energy consumed commercially, the unit of energy, i.e. joule is not sufficient. So, to express electrical energy consumed commercially, a special unit called kilowatt hour is used in place of joule.

1kWh is also called 1 unit of electrical energy. 1 kilowatt hour or 1 unit of electrical energy is the amount of energy dissipated in 1 hour in a circuit, when the electric power in the circuit is 1 kilowatt.

$$1 \text{ kilowatt hour (kWh)} = 3.6 \times 10^6 \text{ joule (J)}$$

EXAMPLE |1| A resistance coil is made by joining in parallel two resistances each of 10 Ω. An emf of 1V is applied between the two ends of coil for 5 min. Calculate the heat produced in calories.

Sol. Given, resistance, $R_1 = 10\Omega$, $R_2 = 10\Omega$

Voltage, $V = 1\text{V}$

and time, $t = 5$ min

$= 5 \times 60$ s $= 300$ s

Since, effective resistance in parallel combination will be

$$R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{10 \times 10}{10 + 10} = 5\Omega$$

$$\therefore \text{Heat produced} = \frac{V^2t}{R_p} = \frac{1^2}{5} \times 5 \times 60 = \frac{60}{4.2} = 14.3 \text{ cal}$$

Electrical Power

It is defined as the rate of electrical energy supplied per unit time to maintain flow of electric current through a conductor.

Mathematically, $\boxed{P = VI = I^2R = \frac{V^2}{R}}$

The SI unit of power is watt (W).

where, 1 watt = 1 volt × 1 ampere = 1 ampere-volt.

Power of an electric circuit is said to be **one watt**, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW).

where, 1kW = 1000 W and 1 MW = 10^6 W

Commercial unit of electrical power is horse power (HP), where 1HP = 746 W.

EXAMPLE |2| A heating element is marked 210 V, 630 W. What is the value of the current drawn by the element when connected to a 210 V DC source?

Delhi 2013

Sol. Given, $P = 630\text{W}$ and $V = 210\text{V}$

Since, $P = VI$

Therefore, $I = \frac{P}{V} = \frac{630}{210} = 3\text{A}$

|TOPIC PRACTICE 2|

OBJECTIVE Type Questions

1. A television of 200 W is used for 4h, then what is the value unit expense of electricity?

(a) 50 (b) 20 (c) 0.8 (d) 0.2

2. Two bulbs of 40W and 60W are connected to 220V line, the ratio of resistance will be

(a) 4 : 3 (b) 3 : 4 (c) 2 : 3 (d) 3 : 2

3. A 100 W-220 V bulb is connected to a supply of 110 V. The power dissipated in the bulb will be
(a) 100 W (b) 50 W
(c) 25 W (d) 2 W

VERY SHORT ANSWER Type Questions

4. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer. **All India 2017**

5. Name the unit of electric energy used for domestic purpose.

6. What is the commercial unit of electrical energy and how is it related to joules?

SHORT ANSWER Type Questions

7. The potential difference applied across a given resistor is altered, so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change? **All India 2017**

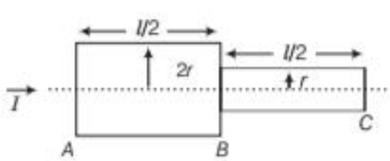
8. Power P is to be delivered to a device *via* transmission cables having resistance R_c. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced. **NCERT Exemplar**

9. When is more power delivered to a light bulb, just after it is turned on and the glow of the filament is increasing or after it has been ON for a few seconds and the glow is steady?

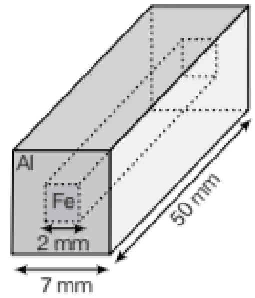
10. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs. **CBSE 2018**

NUMERICAL PROBLEMS

11. Two bars of radius r and $2r$ are kept in contact as shown in the figure. An electric current I is passed through the bars. Find the ratio of heat produced in bars AB and BC.



12. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$-m and $1.0 \times 10^{-7}\ \Omega$-m, respectively.



Calculate the electrical resistance between the two faces P and Q of the composite bar.

13. A room has AC run for 5 hour a day at a voltage of 220 V. The wiring of the room consists of Cu of 1 mm radius and a length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires? What would happen, if the wiring is made of aluminium of the same dimensions?
[Given, $\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega$-m, $\rho_{Al} = 2.7 \times 10^{-8}\ \Omega$-m] **NCERT Exemplar**

HINTS AND SOLUTIONS

1. (c) Dissipated energy in per second,

$$P = \frac{W}{t}$$

$$W = P \times t$$

where, $P = 200$ W, $t = 4$ h

$\Rightarrow$ $W = 200 \times 4$ W-h

Unit of dissipated energy

$$= \frac{\text{watt} \times \text{hours}}{1000}$$

$$= \frac{200 \times 4}{1000} = 0.8 \text{ unit}$$

2. (d) Power, $P = \dfrac{V^2}{R}$

Given, $P_1 = 40$ W, $P_2 = 60$ W

$\therefore$ $$40 = \frac{V^2}{R_1} \quad \text{...(i)}$$

and $$60 = \frac{V^2}{R_2} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{40}{60} = \frac{R_2}{R_1} \text{ or } \frac{R_1}{R_2} = \frac{3}{2} = 3:2$$

3. (b) As we know, $P = \frac{V^2}{R}$ or $P = V \times I$

For 100 W bulb, $100 = 220 \times I \Rightarrow I = \frac{100}{220} = \frac{10}{22}$ A

Hence, the power dissipated for 100W bulb will be

$$P = V \times I = 110 \times \frac{10}{22} = 50 \text{ W}$$

4. For same length and same radius, resistance of wire,

$$R \propto \rho \quad \text{(where } \rho \text{ is resistivity)}$$

As $\quad \rho_{\text{nichrome}} > \rho_{\text{copper}}$

Hence, resistance of nichrome section is more.
In series, same current flows through both sections and heat produced $= I^2Rt$. So, more heat is produced in nichrome section of wire.

5. The unit of electric energy used for domestic purpose is kilowatt hour (kWh). It is also called commercial unit of electric energy.

6. The commercial unit of electrical energy is kilowatt hour (kWh).

$1 \text{kWh} = 3.6 \times 10^6 \text{J}$

7. Heat produced per second $= I^2R = \frac{V^2}{R}$.

So, when voltage is made three times, then heat produced increase nine times for same R.

8. The power consumption in transmission lines is given by $P = i^2R_c$, where R_c is the resistance of transmission lines. The power is given by $\quad P = VI$.

The given power can be transmitted in two ways namely

(i) At low voltage and high current.

(ii) At high voltage and low current.

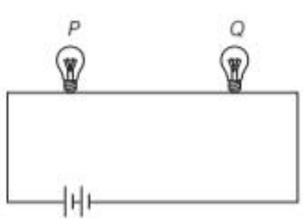
In power transmission at low voltage and high current, more power is wasted as $P \propto i^2$, whereas power transmission at high voltage and low current facilitates the power transmission with minimum power wastage.Thus, power wastage can be reduced by transmitting power at high voltage.

9. When the bulb is turned ON, the resistance of the filament is low, the current is high and a relatively large amount of power is delivered to the bulb.

As the filament warmup, its resistance increases and the current decreases. As a result, power delivered to bulb decreases.

10. Given, $\frac{R_P}{R_Q} = \frac{1}{2}$

$$\therefore \quad R_Q = 2R_P \quad \text{... (i)}$$



In series, power dissipated is given by the relation

$$P = I^2R$$

or $$P \propto R$$

$$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{R_Q} \quad \text{... (ii)}$$

Using Eqs. (i) and (ii), we get

$$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{2R_P} = \frac{1}{2}$$

11. Current flowing through both the bars is equal.

Now, the heat produced is given by

$$E = I^2Rt$$

$$\because \quad E \propto R$$

$$\therefore \quad \frac{E_{AB}}{E_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{(1/2r)^2}{(1/r)^2} \quad \left[\because R \propto \frac{1}{A} \propto \frac{1}{r^2}\right]$$

$$= \frac{1}{4}$$

12. Resistance between the two faces P and Q of the composite bar is given by

$$\frac{1}{R} = \frac{1}{R_{Al}} + \frac{1}{R_{Fe}} = \left(\frac{A_{Al}}{\rho_{Al}} + \frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{l}$$

$$\Rightarrow \quad \frac{1}{R} = \left[\frac{(7^2 - 2^2)}{2.7} + \frac{2^2}{10}\right]\frac{10^{-6}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

$$\therefore \quad R = \frac{1875}{64} \times 10^{-6} \ \Omega = \frac{1875}{64} \ \mu\Omega$$

13. Power consumption in a day, i.e. in 5 h = 10 units

or power consumption per hour = 2 units

or power consumption = 2 units = 2kW = 2000 J/s

Also, we know that, power consumption in resistor,

$$P = V \times I \Rightarrow 2000 \text{ W} = 220 \text{ V} \times I$$

or $$I = 9 \text{ A}$$

Now, the resistance of wire is given by $R = \rho \frac{l}{A}$

where, A is cross-sectional area of conductor.

Power consumption in first current carrying wire is given by $P = I^2R$

$$= \rho \frac{l}{A} I^2 = 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81 \text{ J/s} = 4 \text{ J/s}$$

The fractional loss due to the joule heating in first wire

$$= \frac{4}{2000} \times 100 = 0.2\%$$

Power loss in aluminium wire $= 4 \frac{\rho_{Al}}{\rho_{Cu}} = 1.6 \times 4 = 6.4$ J/s

The fractional loss due to the joule heating in second wire

$$= \frac{6.4}{2000} \times 100 = 0.32\%$$

|TOPIC 3|
Cells, EMF and Internal Resistance

CELLS

An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. Electric cell changes chemical energy into electrical energy.

Electromotive Force (EMF) of a Cell (E)

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the **electromotive force** (**emf**) of the cell.

If during the flow of q coulomb of charge in an electric circuit, the work done by the cell is W, then

emf of the cell, $\boxed{E = \frac{W}{q}}$

Its unit is joule/coulomb or volt.

If $W = 1$ joule and $q = 1$ coulomb, then $E = 1$ volt, i.e. if in the flow of 1 coulomb of charge, the work done by the cell is 1 joule, then the emf of the cell is 1 volt.

Internal Resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte of the cell to the flow of current through it. It is denoted by r. Its unit is ohm.

Internal resistance of a cell depends on the following factors

(i) It is directly proportional to the separation between the two plates of the cell.

(ii) It is inversely proportional to area of plate dipped into the electrolyte.

(iii) It depends on the nature, concentration and temperature of the electrolyte and increases with increase in concentration.

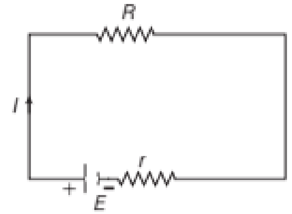
Terminal Potential Difference (V)

Terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (i.e. when current is drawn from the cell). It is represented by V and its unit is volt.

Terminal potential difference of a cell is always less than the emf of the cell. In closed circuit, the current flows through the circuit including the cell, due to internal resistance of the cell there is some fall of potential. This is the amount of potential by which the terminal potential difference is less than the emf of the cell.

Relation between Terminal Potential Difference, emf of a Cell and Internal Resistance of a Cell

(i) If no current is drawn from the cell, i.e. the cell is in open circuit, so emf of the cell will be equal to the terminal potential difference of the cell.



$$I = 0 \quad \text{or} \quad V = E$$

(ii) Consider a cell of emf E and internal resistance r is connected across an external resistance R.

Current drawn from the cell,

$$I = \frac{E}{R + r} \qquad \ldots\text{(i)}$$

where, E = emf of the cell,

R = external resistance

and r = internal resistance of a cell.

Now, from Ohm's law, $V = IR$

$$\Rightarrow \quad I = \frac{V}{R} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{V}{R} = \frac{E}{R + r}$$

$$\Rightarrow \quad \boxed{r = \left(\frac{E}{V} - 1\right)R}$$

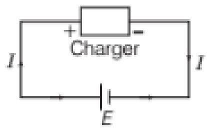
From definition of terminal potential difference,

$$\boxed{V = E - Ir}$$

Charging of a Cell

During charging of a cell, the positive terminal (electrode) of the cell is connected to positive terminal of battery charger and negative terminal (electrode) of the cell is connected to negative terminal of battery charger. In this process, current flows from positive electrode to negative electrode of the cell.

From the given figure, $V = E + Ir$



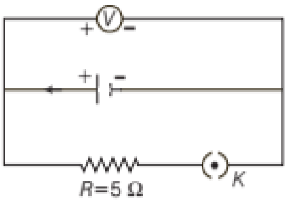
Thus, the terminal potential difference of a cell becomes greater than the emf of the cell.

The potential drop across internal resistance of the cell is called **lost voltage**, as it is not indicated by a voltmeter. Its value is equal to Ir.

Difference between EMF and Terminal Potential Difference of a Cell

S.No.	EMF	Terminal potential difference
1.	The emf of a cell is the maximum potential difference between the two electrodes (terminals) of a cell, when the cell is in the open circuit.	The terminal potential difference of a cell is the potential difference between the two terminals of the cell in a closed circuit.
2.	It is independent of the resistance of the circuit and depends upon the nature of electrodes and electrolyte of the cell.	It depends upon the resistance of the circuit and current flowing through it.
3.	The term emf is used for the source of electric current.	The potential difference is measured between any two points of the electric circuit.
4.	The emf is a cause.	The potential difference is an effect.

EXAMPLE |1| The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2 V. When the terminals of the cell are connected to a resistance of 5 Ω as shown in figure given below, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.



Sol. Given, emf, $E = 2.2\ V$

Terminal potential difference, $V = 1.8\ V$

External resistance, $R = 5\Omega$

$\therefore$ Internal resistance,

$$r = \left(\frac{E-V}{V}\right)R = \left(\frac{2.2-1.8}{1.8}\right)\times 5$$

$$= \frac{10}{9}\ \Omega$$

EXAMPLE |2| A cell of emf E and internal resistance r gives a current of 0.5 A with an external resistance of 12 Ω and a current of 0.25 A with an external resistance of 25 Ω. Calculate the

(i) internal resistance of the cell (ii) emf of the cell.

Sol. Let R be external resistance in series with the cell of emf E and internal resistance r. The current in circuit is

$$I = \frac{E}{R+r}$$

***Case* I** $I = 0.5\text{ A},\ R = 12\ \Omega$, then

$$0.5 = \frac{E}{12+r}$$

$$\Rightarrow \quad E = 0.5\,(12+r)$$

$$\Rightarrow \quad E = 6.0 + 0.5r \qquad \text{...(i)}$$

***Case* II** $I = 0.25\text{ A},\ R = 25\Omega$, then

$$0.25 = \frac{E}{25+r}$$

$$\Rightarrow \quad E = 0.25\,(25+r)$$

$$\Rightarrow \quad E = 6.25 + 0.25r \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$6.0 + 0.5\ r = 6.25 + 0.25\ r$$

$$\Rightarrow \quad r = 1\Omega$$

From Eq. (i), we get

$$E = 6.0 + 0.5 \times (1) = 6.5\text{V}$$

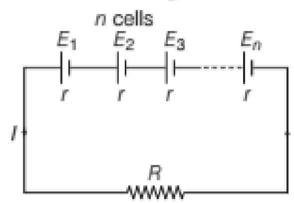
Hence, (i) internal resistance of the cell is 1 Ω.

(ii) emf of the cell is 6.5 V.

CELLS IN SERIES AND PARALLEL

Cells in Series

In this combination, n identical cells each of emf E and internal resistance r are connected in series to the external resistance R as shown in the figure.



Points to remember for series combination of cells

(i) The equivalent emf of a series combination of n cells is equal to the sum of their individual emfs.

(ii) The equivalent internal resistance of a series combination of n cells is equal to sum of their individual internal resistances.

Equivalent emf of n cells in series,

$E_{eq} = E_1 + E_2 + \ldots +$ upto n terms $= nE$

Equivalent internal resistance of n cells in series,

$r_{eq} = r_1 + r_2 + \ldots +$ upto n terms $= nr$

Total resistance of the circuit $= nr + R$

$\therefore$ Current in the resistance R is given by

$$I = \frac{nE}{R + nr}$$

where, n = number of cells,
r = internal resistance,
R = external resistance,
E = emf of cell.
and I = current flowing.

Case I When $R << nr$, then

$I = \frac{E}{r}$ = current due to a single cell

Case II When $R >> nr$, then

$I = \frac{nE}{R} = n$ times the current due to a single cell

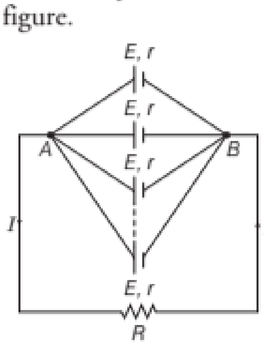
Case III When cells are of different emfs and different internal resistances, then

$$I = \frac{E_1 + E_2 + \ldots + E_n}{R + (r_1 + r_2 + \ldots + r_n)}.$$

Note The maximum current can be drawn from the series combination of cells, if the value of external resistance is very high as compared to the total internal resistance of the cells.

Cells in Parallel

In this combination, m cells each of emf E and internal resistance r are connected in parallel the external resistance R as shown in the figure.



Points to remember for parallel combination of cells

(i) The equivalent emf of parallel combination of cells of same emfs is equal to emf of one cell.

(ii) The reciprocal of equivalent internal resistance of parallel combination of cells is equal to the sum of the reciprocals of the internal resistance of each cell.

$\therefore \frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots$ upto m terms $= \frac{m}{r}$ or $r_p = \frac{r}{m}$

As, R and r_p are in series, so total resistance in the circuit $= R + \frac{r}{m}$

In parallel combination of identical cells, the effective emf in the circuit is equal to the emf due to a single cell, because in this combination, only the size of the electrodes increases but not emf.

$\therefore$ Current in the resistance R is given by

$$I = \frac{E}{R + \frac{r}{m}}$$

Case I When $R >> \frac{r}{m}$, then

$I = \frac{E}{R}$ = current due to a single cell

Case II When $R << \frac{r}{m}$, then $I = \frac{E}{r/m}$

$= \frac{mE}{r} = m$ times current due to a single cell

Case III When cells are of same emf and different internal resistances, then

$I = \frac{E}{R + r'}$ $\quad [\because E_1 = E_2 = \ldots E_n = E]$

where, $\frac{1}{r'} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_n}$ and E is emf of each cell.

Note The maximum current can be drawn from the parallel combination of cells, if the external resistance is very low as compared to the total internal resistance of the cells.

EXAMPLE |3| Two identical cells, when joined together in series or in parallel give the same current, when connected to external resistance of 2 Ω. Find the internal resistance of each cell.

Sol. Let E, r be the emf and internal resistance of each cell.

External resistance, $R = 2\Omega$

If two cells are connected in series, then

Total emf of cells $= E + E = 2E$

Total resistance of circuit $= R + r + r = 2 + 2r$

Current in the circuit, $I_1 = \dfrac{2E}{2+2r}$

If two cells are connected in parallel, effective emf of two cells = emf of single cell = E

Total internal resistance of two cells $= \dfrac{r \times r}{r+r} = \dfrac{r}{2}$

Total resistance of the circuit $= R + \dfrac{r}{2} = 2 + \dfrac{r}{2}$

Current in the circuit, $I_2 = \dfrac{E}{2+\dfrac{r}{2}} = \dfrac{2E}{4+r}$

As per question, $I_1 = I_2$

$$\Rightarrow \quad \frac{2E}{2+2r} = \frac{2E}{4+r}$$

$$\Rightarrow \quad 2 + 2r = 4 + r$$

$$\therefore \quad r = 2\Omega$$

EXAMPLE |4| When 14 cells in series, are connected to the ends of a resistance of 82.6 Ω, then the current is found to be 0.25A. When same cells after being connected in parallel are joined to the ends of a resistance of 0.053Ω, then the current is 25A. Calculate the internal resistance and the emf of each cell.

Sol. Let E and r be the emf and internal resistance of each cell.

***Case* I** When the cells are in series.

Total emf of cells $= 14E$

Total resistance of circuit $= 82.6 + 14r$

$\therefore$ Current in the circuit is given by

$$\frac{14\,E}{82.6+14r} = 0.25\,\text{A} \qquad \text{...(i)}$$

***Case* II** When the cells are in parallel.

Total emf of cells $= E$

Total resistance of circuit $= 0.053 + \dfrac{r}{14}$

$\therefore$ Current in the circuit is given by

$$\frac{E}{0.053+\dfrac{r}{14}} = 25\,\text{A} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$14\frac{\left(0.053+\dfrac{r}{14}\right)}{(82.6+14r)} = 10^{-2}$$

$$\Rightarrow \quad 14 \times \frac{14 \times 0.053 + r}{14} \times 10^2 = 82.6 + 14r$$

$$\Rightarrow \quad 5.3 \times 14 + 100r = 82.6 + 14r$$

Solving, we get

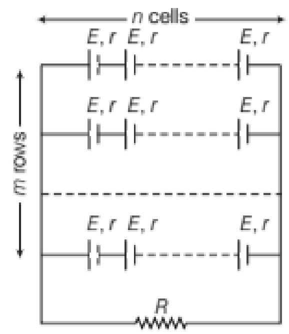
$$r = 0.097\Omega = 0.1\,\Omega$$

Substituting the value of r in Eq. (i), we get

$$E = 1.5\,\text{V}$$

Mixed Combination of Cells

In this combination, some cells are connected in series and some cells are connected in parallel as shown in the figure.



Let there be n cells in series in one row and m rows of cells are in parallel.

Suppose all the cells are identical. Let each cell be of emf and internal resistance r.

Equivalent emf of each row $= nE$

Equivalent internal resistance of each row $= nr$

Total emf of combination $= nE$

Total internal resistance of combination,

$$\frac{1}{r'} = \frac{1}{nr} + \frac{1}{nr} + \dots \text{ upto } m \text{ times}$$

$$\frac{1}{r'} = \frac{m}{nr} \text{ or } r' = \frac{nr}{m}$$

Total resistance of the circuit $= r' + R = \dfrac{nr}{m} + R$

Current in the resistance R is given by

$$\boxed{I = \frac{nE}{\dfrac{nr}{m} + R}}$$

Thus, we get the maximum current in mixed grouping of cells, if the value of external resistance is equal to the total internal resistance of all the cells, i.e. external resistance = total internal resistance of all the cells $\left(R = \dfrac{nr}{m}\right)$.

EXAMPLE |5| 36 cells, each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external circuit of 2 Ω resistance. Find the best mode of grouping them and the current through the external circuit.

Sol. Here, $E = 1.5\,\text{V}$, $r = 0.5\,\Omega$, $R = 2\,\Omega$

Total number of cells, $mn = 36$ ...(i)

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R$$

$$\Rightarrow \qquad \frac{n \times 0.5}{m} = 2 \qquad \text{... (ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$0.5n^2 = 72 \Rightarrow n^2 = 144$$

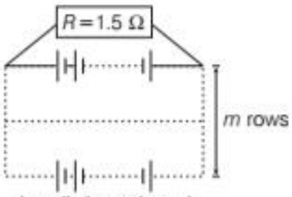
$$\therefore \qquad n = 12$$

and $$m = \frac{36}{12} = 3$$

Thus, for maximum current, there should be three rows in parallel, each containing 12 cells in series.

$$\therefore \text{ Maximum current} = \frac{mnE}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 0.5} = 4.5 \text{ A}$$

EXAMPLE |6| 12 cells, each of emf 1.5 V and internal resistance of 0.5 Ω, are arranged in m rows each containing n cells connected in series, as shown in the figure. Calculate the values of n and m for which this combination would send maximum current through an external resistance of 1.5 Ω.



(n cells in each row)

Sol. For maximum current through the external resistance, external resistance = total internal resistance of cells

or $$R = \frac{nr}{m}$$

$$\therefore \qquad 1.5 = \frac{n \times 0.5}{\frac{12}{n}} \qquad [\because mn = 12]$$

or $$36 = n^2$$

$$\therefore \qquad n = 6 \text{ and } m = 2$$

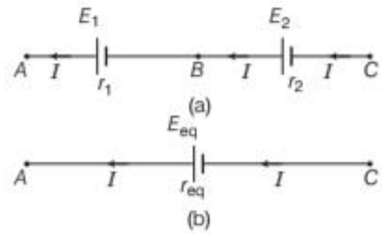
COMBINATION OF TWO CELLS IN SERIES AND PARALLEL

(WITH DIFFERENT EMFS AND INTERNAL RESISTANCES)

Two Cells in Series

The two cells are said to be connected in series between two points A and C, when negative terminal of one cell is connected to positive terminal of other cell as shown in the Fig. (a).

Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively. Let the two cells be sending the current in a circuit shown in the Fig. (a) and (b). Let V_A, V_B and V_C be the potentials at points A, B and C and I be the current flowing through them.



Potential difference between positive and negative terminals of the first cell is given by

$$V_{AB} = V_A - V_B = E_1 - Ir_1 \qquad \text{...(i)}$$

Potential difference between positive and negative terminals of second cell is given by

$$V_{BC} = V_B - V_C = E_2 - Ir_2 \qquad \text{...(ii)}$$

Potential difference between A and C of the series combination of the two cells is given by

$$\begin{aligned} V_{AC} &= V_A - V_C \\ &= (V_A - V_B) + (V_B - V_C) \\ &= (E_1 - Ir_1) + (E_2 - Ir_2) \\ &= (E_1 + E_2) - I(r_1 + r_2) \qquad \text{...(iii)} \end{aligned}$$

If the series combination of two cells is replaced by single cell between A and C of emf E_{eq} and internal resistance r_{eq} as shown in the Fig. (b), then

$$V_{AC} = E_{eq} - I\,r_{eq} \qquad \text{...(iv)}$$

Comparing Eqs. (iii) and (iv), we get

$$E_{eq} = E_1 + E_2 \qquad \text{...(v)}$$

and $$r_{eq} = r_1 + r_2 \qquad \text{...(vi)}$$

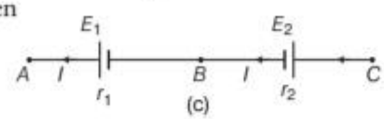
If n cells of emfs $E_1, E_2 \ldots E_n$ and of internal resistances $r_1, r_2, \ldots, r_n$ respectively, are connected in series between points A and C, then equivalent emf is given by

$$\boxed{E_{eq} = E_1 + E_2 + \ldots + E_n}$$

Equivalent internal resistance of the cells is given by

$$\boxed{r_{eq} = r_1 + r_2 + \ldots + r_n}$$

That in the series combination of two cells, if negative terminal of first cell is connected to the negative terminal of the second cell between points A and C, as shown in the Fig. (c), then

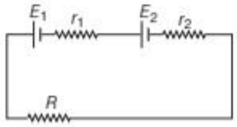


$$V_{BC} = V_B - V_C = -E_2 - Ir_2$$

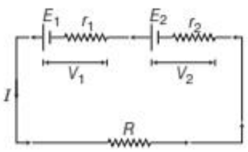
Then, equivalent emf of the two cells is $E_{eq} = E_1 - E_2$

But equivalent internal resistance is $r_{eq} = r_1 + r_2$.

EXAMPLE |7| In the circuit shown in figure, $E_1 = 10$ V, $E_2 = 4$ V, $r_1 = r_2 = 1\,\Omega$ and $R = 2\,\Omega$. Find the potential difference across battery 1 and battery 2.



Sol. Net emf of the circuit $= E_1 - E_2 = (10-4) = 6$ V



Total resistance of the circuit $= R + r_1 + r_2 = 4\,\Omega$

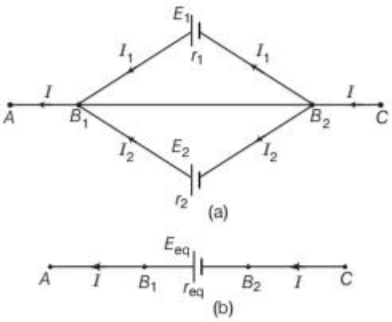
$\therefore$ Current in the circuit, $I = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{6}{3} = 1.5$ A

Now, $V_1 = E_1 - Ir_1 = 10 - (1.5)(1) = 8.5$ V

and $V_2 = E_2 + Ir_2 = 4 + (1.5)(1) = 5.5$ V

Two Cells in Parallel

The two cells are said to be connected in parallel between two points A and C, when positive terminal of each cell is connected to one point and negative terminal of each cell is connected to the other point as shown in the Fig. (a).



Let the two cells be sending the current in a circuit shown in Figs. (a) and (b). Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively.

Let I_1, I_2 be the currents from the two cells flowing towards point B_1 and I be the current flowing out of B_1, then

$$I = I_1 + I_2 \qquad \ldots\text{(i)}$$

Let V_{B_1}, V_{B_2} be the potentials at points B_1 and B_2, respectively and V be the potential difference between B_1 and B_2. Here, the potential difference across the terminals of first cell is equal to the potential difference across the terminals of the second cell.

So, for the first cell,

V is given by $V = V_{B_1} - V_{B_2} = E_1 - I_1 r_1$ or $I_1 = \dfrac{E_1 - V}{r_1}$

For the second cell, $V = V_{B_1} - V_{B_2} = E_2 - I_2 r_2$

or $$I_2 = \frac{E_2 - V}{r_2}$$

Substituting values in Eq. (i), we get

$$I = \left(\frac{E_1 - V}{r_1}\right) + \left(\frac{E_2 - V}{r_2}\right)$$

$$= \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$\Rightarrow \boxed{V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - \frac{I\, r_1 r_2}{r_1 + r_2}} \qquad \ldots\text{(ii)}$$

If the parallel combination of cells is replaced by a single cell between B_1 and B_2 of emf E_{eq} and internal resistance r_{eq} [Fig. (b)], then

$$V = E_{eq} - Ir_{eq} \qquad \ldots\text{(iii)}$$

Comparing Eqs. (ii) and (iii), we get

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \qquad \ldots\text{(iv)}$$

and $$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \qquad \ldots\text{(v)}$$

$$\Rightarrow \frac{1}{r_{eq}} = \frac{r_1 + r_2}{r_1 r_2} = \frac{1}{r_1} + \frac{1}{r_2} \qquad \ldots\text{(vi)}$$

Dividing Eq. (iv) by Eq.(v), we get

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

If n cells of emfs $E_1, E_2, \ldots, E_n$ and internal resistances $r_1, r_2, \ldots, r_n$ are connected in parallel, whose equivalent emf is E_{eq} and equivalent internal resistance is r_{eq}, then

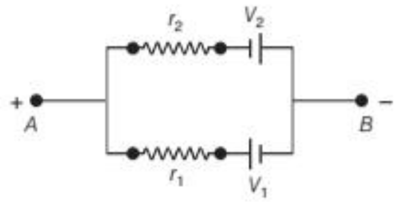
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots + \frac{1}{r_n} \text{ and } \frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots + \frac{E_n}{r_n}$$

If the two cells are connected in parallel and are of the same emf E and same internal resistance r, then

From Eq. (iv), $E_{eq} = \dfrac{E\,r + E\,r}{r + r} = E$

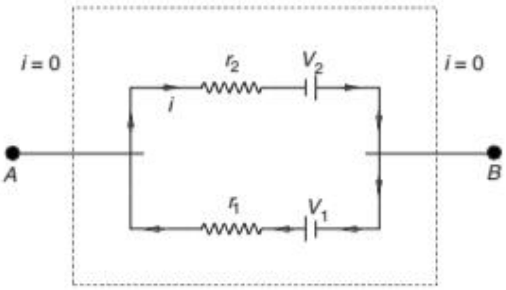
From Eq. (vi), $\dfrac{1}{r_{eq}} = \dfrac{1}{r} + \dfrac{1}{r} = \dfrac{2}{r} \Rightarrow r_{eq} = \dfrac{r}{2}$

EXAMPLE |8| Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistances r_1 and r_2 respectively, with polarities as shown in figure



Sol. (i) **Equivalent emf (V) of the battery**

Potential difference across the terminals of the battery is equal to its emf when current drawn from the battery is zero. In the given circuit,



Current in the internal circuit,

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{V_1 + V_2}{r_1 + r_2}$$

Therefore, potential difference between A and B would be

$$V_A - V_B = V_1 - ir_1$$

$$= V_1 - \left(\frac{V_1 + V_2}{r_1 + r_2}\right) r_1 = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

So, the equivalent emf of the battery is

$$V = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

Note that, if $V_1 r_2 = V_2 r_1 : V = 0$

If $V_1 r_2 > V_2 r_1 : V_A - V_B =$ positive, i.e. A side of the equivalent battery will become the positive terminal and *vice-versa*.

(ii) **Internal resistance (r) of the battery**

r_1 and r_2 are in parallel. Therefore, the internal resistance r will be given by

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\Rightarrow \quad r = \frac{r_1 r_2}{r_1 + r_2}$$

| TOPIC PRACTICE 3 |

OBJECTIVE Type Questions

1. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is **NEET 2013**

(a) 0.2 Ω (b) 0.5 Ω (c) 0.8 Ω (d) 1.0 Ω

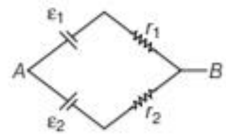
2. The cell has an emf of 2V and the internal resistance of this cell is 0.1 Ω, it is connected to resistance of 3.9 Ω, the voltage across the cell will be

(a) 1.95 V (b) 1.5 V (c) 2 V (d) 1.8 V

3. Electromotive force of primary cell is 2.4 V. When cell is short circuited, then current becomes 4 A. Internal resistance of cell is

(a) 60 Ω (b) 1.2 Ω
(c) 4 Ω (d) 0.6 Ω

4. Two batteries of emf ε_1 and ε_2, ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



NCERT Exemplar

(a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
(b) The equivalent emf ε_{eq} is smaller than ε_1
(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
(d) ε_{eq} is independent of internal resistances r_1 and r_2

VERY SHORT ANSWER Type Questions

5. The emf of a cell is always greater than its terminal voltage. Why? **Delhi 2013**

6. A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential difference across R, V *versus* R. **NCERT Exemplar**

7. Write any two factors on which internal resistance of a cell depends. **All India 2013**

8. A cell of emf E and internal resistance r is connected across a variable load resistor R.

Draw the plots of the terminal voltage V *versus* (i) resistance R and

(ii) current I. **All India 2015**

9. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing variation of terminal voltage V of the cell *versus* the current I. Using the plot, show how the emf of the cell and its internal resistance can be determined. **All India 2014**

10. Two identical cells, each of emf E, having negligible internal resistance are connected in parallel with each other across an external resistance R. What is the current through this resistance? **All India 2013**

11. Which of the two emf E or potential difference V of a cell, is greater and by how much?

SHORT ANSWER Type Questions

12. First a set of n equal resistors of R each are connected in series to a battery of emf E and internal resistance R and current I is observed to flow. Then, the resistors are connected in parallel to the same battery.

It is observed that the current is increased 10 times. What is n? **NCERT Exemplar**

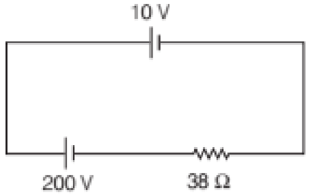
13. Write the relation between emf and potential difference for a cell. What are their respective units?

14. What is the difference between the values of potential difference across the two terminals of a cell is an open circuit and closed circuit?

15. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing the variation of terminal potential V with resistance R. Predict from the graph, the condition under which V becomes equal to E. **Delhi 2009**

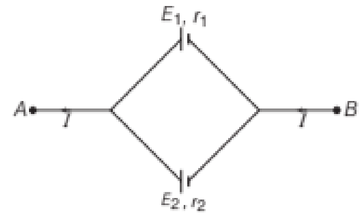
16. A low voltage supply from which one needs high currents must have very low internal resistance. Why?

17. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance 38 Ω as shown in the figure. Find the value of current in the circuit. **CBSE 2018**



LONG ANSWER Type I Questions

18. Two cells of emf E_1 and E_2; and internal resistances r_1 and r_2 respectively, are connected in parallel as shown in the figure.



Deduce the expressions for

(i) the equivalent emf of the combination.

(ii) the equivalent resistance of the combination.

(iii) the potential difference between the points A and B. **Foreign 2010**

19. Which type of combination of cells is used in the following three cases.

(i) If the external resistance is much larger than the total internal resistance?

(ii) If the external resistance is much smaller than the total internal resistance?

(iii) If the external resistance is equal to the total internal resistance?

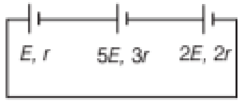
20. What do you mean by terminal potential difference of a cell? Under what conditions will the terminal potential difference of a cell be greater than its emf?

LONG ANSWER Type II Question

21. (i) The emf of a cell is always greater than its terminal voltage. Why? Give reason.

(ii) Plot a graph showing the variation of terminal potential difference across a cell of emf E and internal resistance r with current drawn from it. Using this graph, how does one determine the emf of the cell?

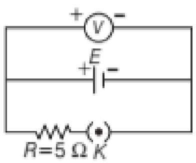
(iii) Three cells of emf $E, 2E$ and $5E$ having internal resistances r, $2r$ and $3r$, variable resistance R as shown in the figure. Find the expression for the current. Plot a graph for variation of current with R.



NUMERICAL PROBLEMS

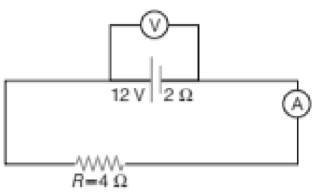
22. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of resistor? What is the terminal voltage of the battery when the circuit is closed? **NCERT**

23. The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2 V. When the terminals of the cell are also connected to a resistance of 5 Ω as shown in the circuit, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell. **All India 2013**



24. It is found that when $R = 4\ \Omega$, the current is 1 A and when R is increased to 9 Ω, the current reduces to 0.5 A. Find the values of the emf E and internal resistance r. **All India 2015**

25. In the figure shown, an ammeter A and a resistor of 4 Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2 Ω. Calculate the voltmeter and ammeter readings.



All India 2017

26. A battery of emf 12 V and internal resistance 2 Ω is connected to a 4 Ω resistor as shown in the figure.

(i) Show that a voltmeter when placed across the cell and across the resistor, in turn, gives the same reading.

(ii) To record the voltage and the current in the circuit, why is voltmeter placed in parallel and ammeter in series in the circuit? **All India 2016**

27. A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of 38 Ω as shown in the figure. Find the value of the current in the circuit. **Delhi 2013**

28. (i) Six lead-acid type of secondary cells each of emf 2 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

(ii) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell?

Could the cell drive the starting motor of a car? **NCERT**

HINTS AND SOLUTIONS

1. (b) As, $I = \dfrac{E}{R+r}$

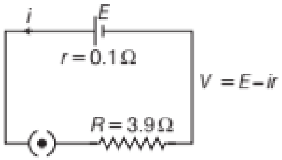
or $E = I(R+r)$

$2.1 = 0.2\,(10+r)$

$10 + r = \dfrac{2.1}{2} \times 10$

$\therefore$ $r = 10.5 - 10 = 0.5\ \Omega$

2. (a)

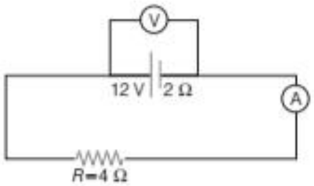


$\therefore$ $V = E - ir$

where, r is the internal resistance.

Also, current $i = \dfrac{E}{R+r}$

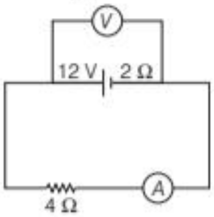
$\Rightarrow$ $V = E - \left(\dfrac{E}{R+r}\right) r$



All India 2017

26. A battery of emf 12 V and internal resistance 2 Ω is connected to a 4 Ω resistor as shown in the figure.

(i) Show that a voltmeter when placed across the cell and across the resistor, in turn, gives the same reading.



$\therefore \quad V = E - ir$

where, r is the internal resistance.

Also, current $i = \dfrac{E}{R+r}$

$\Rightarrow \quad V = E - \left(\dfrac{E}{R+r}\right)r$

Putting numerical values, we have

$E = 2\text{ V}, \quad r = 0.1\ \Omega, R = 3.9\ \Omega$

$\Rightarrow \quad V = 2 - \left(\dfrac{2}{3.9+0.1}\right) \times 0.1$

$\Rightarrow \quad V = 1.95\text{ V}$

3. (d) Electromotive force, $E = V + ir = i(R + r)$. [$\because V = iR$]

When cell is short circuited, then resistance becomes zero, i.e. $R = 0$. So, electromotive force, $E = ir$

Internal resistance of cell

$$r = \frac{E}{i} = \frac{2.4}{4} = 0.6\ \Omega$$

4. (a) The equivalent emf of this combination is given by

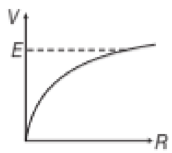
$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 + \varepsilon_1 r_2}{r_1 + r_2}$$

This suggest that the equivalent emf ε_{eq} of the two cells is given by

$$\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$$

5. The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance.

6. The graphical relationship between voltage across R and the resistance R is given below



$$\left[\because V = \frac{E}{1 + r/R}\right]$$

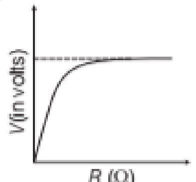
7. The high resistance voltmeter means that current will flow through it. Hence, there is no potential difference across it. So, the reading shown by the high resistance voltmeter can be taken as the emf of the cell.

The internal resistance of a cell depends on

(i) the concentration of electrolyte and

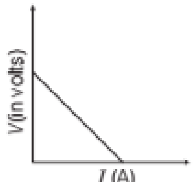
(ii) distance between the two electrodes.

8. (i)



Graph between terminal voltage (V) and resistance (R)

(a)

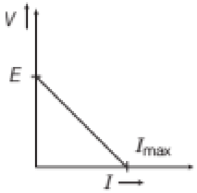


Graph between terminal voltage (V) and current (I)

(b)

9. We know that, $V = E - Ir$

The plot between V and I is a straight line of positive intercept and negative slope as shown below



(i) The value of potential difference corresponding to zero current gives emf of the cell.

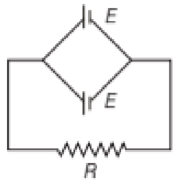
(ii) Maximum current is drawn, when terminal voltage is zero, so

$$V = E - Ir$$

$$\Rightarrow \quad 0 = E - I_{max} r$$

$$\Rightarrow \quad r = \frac{E}{I_{max}}$$

10. The cells are arranged as shown in the circuit diagram as given below



As the internal resistance is negligible, so total resistance of the circuit = R

So, current through the resistance,

$$I = \frac{E}{R}$$

(in parallel combination, potential is same as the single cell)

11. emf E of the cell is greater than the potential difference V of the cell, by a value Ir, where I is the current flowing in the circuit and r is the internal resistance of the cell.

$$V = E - Ir$$

12. In series combination of resistors, current I is given by

$$I = \frac{E}{R + nR}$$

whereas, in parallel combination current 10 I is given by

$$\frac{E}{R + \frac{R}{n}} = 10\,I \Rightarrow \frac{E}{R + \frac{R}{n}} = 10\left(\frac{E}{R + nR}\right)$$

Now, according to problem,

$$\frac{1 + n}{1 + \frac{1}{n}} = 10 \Rightarrow 10 = \left(\frac{1 + n}{n + 1}\right) n \Rightarrow n = 10$$

13. For a cell of emf E, potential difference V and internal resistance r, $V = E - Ir$, where I is the current flowing through the circuit. The SI unit of both emf and potential difference of a cell is volt (V).

14. The potential difference across the terminals of a cell is given by $V = E - Ir$.

In an open circuit, there is no current, i.e. $I = 0$

$\therefore$ $V = E$, i.e. potential difference across the terminals of a cell = emf

In a closed circuit, $V < E$.

The difference between the two values of potential difference = Ir, which is called the lost voltage.

15. Refer to the solution of Q. 6 for the graph.
From the graph, we can see that the value of V becomes equal to E when $I = 0$.

16. We know that, $V = E - Ir$

$\therefore$ Current in the circuit, $I = \frac{E - V}{r}$

If the value of V is small, for high value of current I, then the internal resistance r should be small as $I \propto \frac{1}{r}$.

17. Given, $\varepsilon = 10$ V, $E = 200$ V,

Now, using Kirchhoff's loop law in given figure, in loop $ABCDA$,

$$200 - 38I - 10 = 0$$

$$190 = 38I$$

$$\therefore \quad I = \frac{190}{38} = 5\text{ A}$$

18. Refer to text on page 142.

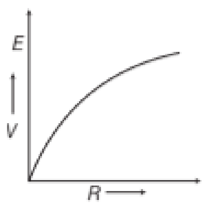
19. (i) Series combination of cells.
(ii) Parallel combination of cells.
(iii) Mixed combination of cells.

20. Refer to text on page 140.
The terminal potential difference of the cell becomes greater than the emf of the cell during charging of the cell. In this process, current flows from positive electrode to negative electrode of the cell.
Hence, $V = E + Ir$.

21. (i) The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance.

(ii) $\because \quad V = \left(\frac{E}{R + r}\right)R = \frac{E}{1 + r/R}$

i.e. with the increase of R, V increases



One can determine the emf of cell by finding terminal potential difference when current I becomes zero.

(iii) In these type of questions, we have to look out the connections of different cells, if the opposite terminals of all the cells are connected, then they support each other, i.e. these individual emf's are added up. If the same terminals of the cells are connected, then the equivalent emf is obtained by taking the difference of emf's.

Net emf of combination

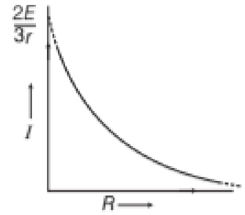
$$= E - 2E + 5E = 4E$$

Net resistance of current

$$= r + 2r + 3r + R = 6r + R$$

$\therefore$ Current, $I = \frac{V}{R}$ (from Ohm's law)

$$\Rightarrow \quad I = \frac{4E}{6r + R}$$



22. Given, $E = 10$ V, $r = 3\Omega$, $I = 0.5$ A

As, $\quad I = \frac{E}{R + r}$

$$\Rightarrow \quad R = \frac{E}{I} - r$$

$$= \frac{10}{0.5} - 3 = 17\,\Omega$$

and terminal voltage, $V = IR = 0.5 \times 17 = 8.5$ V

23. The emf of cell, $E = 2.2$ V

The terminal voltage across cell, when 5 Ω resistance R is connected across it, $V = 1.8$ V

Let internal resistance $= r$

$\therefore$ Internal resistance, $r = R\left(\frac{E}{V} - 1\right)$

$$= 5\left(\frac{2.2}{1.8} - 1\right)$$

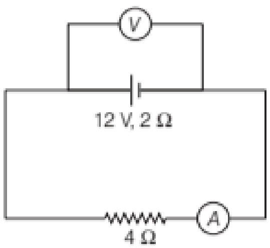
$$= 5 \times \frac{0.4}{1.8} = \frac{2}{1.8} = \frac{10}{9}\,\Omega$$

24. Refer to Example 2 on page 141. **[Ans.** 1 Ω and 5 V]

25. Current in the circuit, $I = \frac{E}{R + r} = \frac{12}{4 + 2} = 2$ A

Also, terminal voltage across the cell,
$V = E - Ir = 12 - 2 \times 2 = 8$ V
So, ammeter reading = 2 A
and voltmeter reading = 8 V

26. According to question,



(i) Net current in the circuit $= \frac{12}{6} = 2$ A

Voltage across the battery,
$$V_b = 12 - 2 \times 2 = 8\text{V}$$

Voltage across the resistance,

$$V_r = IR = 2 \times 4 = 8\text{V}$$

(ii) In order to measure the device's voltage for a voltmeter, it must be connected in parallel to that device. This is necessary because device in parallel experiences the same potential difference. An ammeter is connected in series with the circuit because the purpose of the ammeter is to measure the current through the circuit. Since, the ammeter is a low impedance device. Connecting in parallel with the circuit would cause a short circuit, damaging the ammeter of the circuit.

27. Since, the positive terminal of the batteries are connected together, so the equivalent emf of the batteries is given by

$$E = 200 - 10 = 190 \text{ V}$$

Hence, the current in the circuit is given by

$$I = \frac{E}{R} = \frac{190}{38} = 5 \text{ A}$$

28. (i) Six cells are joined in series.

emf of each cell, $E = 2$ V

Number of cells, $n = 6$

Total emf of circuit $= n \times E = 6 \times 2 = 12$ V

Internal resistance of each cell, $r = 0.015\ \Omega$

Total internal resistance

$$= n \times r = 6 \times 0.015 = 0.09\ \Omega$$

External load, $R = 8.5\ \Omega$

Current in the circuit, $I = \dfrac{nE}{nr + R}$

$$= \frac{12}{0.09 + 8.5} = 1.4 \text{ A}$$

$\therefore$ The terminal voltage of battery,

$$V = IR = 1.4 \times 8.5 = 11.9 \text{ V}$$

(ii) emf of cell, $E = 1.9$ V

Internal resistance of cell,

$$r = 380\ \Omega$$

Maximum current can be drawn from the cell, if there is zero external resistance. Therefore,

$$I_{\max} = \frac{E}{r} = \frac{1.9}{380} = 0.005 \text{ A}$$

Now, we see that the maximum current drawn from the cell is very low, thus the cell cannot be used to drive the starting motor of a car as the current required for this purpose is approximately 100 A for few records.

|TOPIC 4|

Kirchhoff's Laws and its Applications

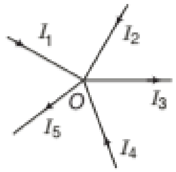
KIRCHHOFF'S RULES OR LAWS

In 1842, Kirchhoff gave the following two rules to solve complicated electrical circuits. Ohm's law is simply not adequate for the study of the circuits containing more than one source of emf. These rules are basically the expressions of conservation of electric charge and energy.

These laws were stated as follows

First Law (Junction Rule)

This law states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero. It is also known as junction rule.



Electric junction

Consider a point O in an electrical circuit at which currents I_1, I_2, I_3, I_4 and I_5 flowing through the different conductors meet, as shown in the figure.

According to Kirchhoff's first law, we have

$$I_1 + I_2 + (-I_3) + I_4 + (-I_5) = 0$$

$$\Rightarrow \quad I_1 + I_2 - I_3 + I_4 - I_5 = 0$$

$$\therefore \quad I_1 + I_2 + I_4 = I_3 + I_5$$

So, junction rule can also be stated as the sum of currents entering the junction is equal to the sum of currents leaving the junction.

Sign Convention for Kirchhoff's First Law

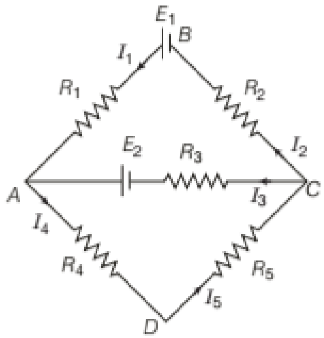
The current flowing towards the junction of conductors is considered as positive and the current flowing away from the junction is taken as negative.

Second Law

(Kirchhoff's Voltage Rule)

This law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. It means that in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as **loop rule**.

Consider a closed electrical circuit $ABCDA$ containing two cells E_1 and E_2 and five resistances R_1, R_2, R_3, R_4 and R_5.



Consider the closed loop $ABCA$. E_1 will send current in anti-clockwise and E_2 will send current in clockwise direction.

$\therefore$ Total emf of closed loop

$$ABCA = E_1 + (-E_2) = E_1 - E_2$$

But currents (I_1 and I_2) flow in anti-clockwise direction while current I_3 flows in clockwise direction.

The algebraic sum of products of resistances and current

$$= I_1R_1 + I_2R_2 + (-I_3)R_3$$
$$= I_1R_1 + I_2R_2 - I_3R_3$$

$\therefore$ According to second law, for closed part $ABCA$,

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

Similarly, for closed part $ACDA$,

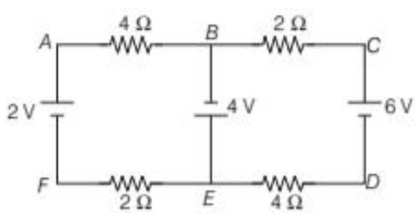
$$E_2 = I_3R_3 + I_4R_4 + I_5R_5$$

Sign Convention for Kirchhoff 's Second Law

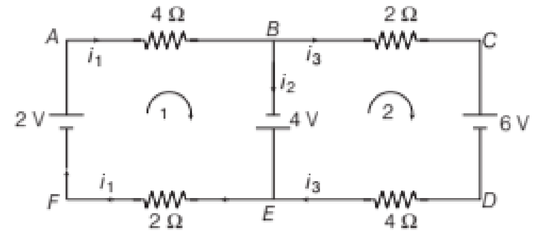
The product of resistance and current in an arm of the loop is taken as positive, if the direction of current in that arm is in the same sense as one moves and is taken as negative, if the direction of current in an arm is opposite to the sense as one moves.

While traversing a loop, the emf of a cell is taken negative, if negative pole of the cell is encountered first, otherwise positive.

EXAMPLE |1| Find currents in different branches of the electric circuit shown in figure.



Sol.



Applying Kirchhoff's first law (junction law) at junction B,

$$i_1 = i_2 + i_3 \quad \text{...(i)}$$

Applying Kirchhoff's second law in loop 1 ($ABEFA$),

$$-4i_1 + 4 - 2i_1 + 2 = 0 \quad \text{...(ii)}$$

Applying Kirchhoff's second law in loop 2 ($BCDEB$),

$$-2i_3 - 6 - 4i_3 - 4 = 0 \quad \text{...(iii)}$$

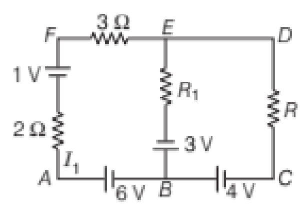
Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1\text{ A}$$

$$\Rightarrow \quad i_2 = \frac{8}{3}\text{ A} \Rightarrow i_3 = -\frac{5}{3}\text{ A}$$

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

EXAMPLE |2| Use Kirchhoff's rules to determine the potential difference between the points A and D. When no current flows in the arm BE of the electric network shown in the figure below. **Delhi 2015**



Sol. Applying Kirchhoff's loop rule for loop $ABEFA$

$$6 + 3 + R_1 \times 0 - 3I_1 + 1 - 2I_1 = 0$$

or $$10 - 5I_1 = 0$$

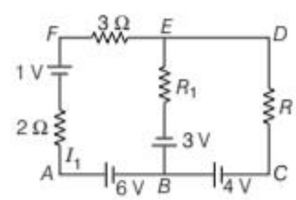
or $$I_1 = 2\text{A}$$

For loop $BCDEB$,

$$4 - I_1 \cdot R + R_1 \times 0 - 3 = 0$$

or $$1 - 2R = 0$$

$\therefore$ $$R = \frac{1}{2}\,\Omega$$



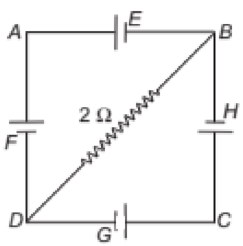
Potential difference between A and D through path $ABCD$ is

$$6+4-I_1R=V_{AD}$$

$$\Rightarrow \quad 10-2\times\frac{1}{2}=V_{AD}$$

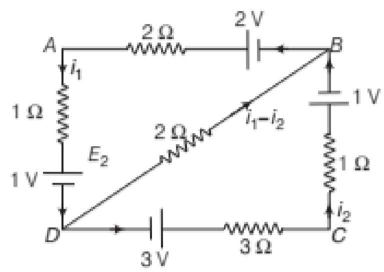
$$\therefore \quad V_{AD}=9 \text{ volt}$$

EXAMPLE |3| In the circuit shown in figure E, F, G, H are cells of emf 2, 1, 3 and 1 V respectively, and their internal resistances are 2, 1, 3 and 1 Ω, respectively. Calculate



(i) the potential difference between B and D and
(ii) the potential difference across the terminals of each cells G and H.

Sol.



Applying Kirchhoff's second law in loop $BADB$,

$$2-2i_1-i_1-1-2(i_1-i_2)=0 \quad \text{...(i)}$$

Similarly, applying Kirchhoff's second law in loop $BDCB$,

$$2(i_1-i_2)+3-3i_2-i_2-1=0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$i_1=\frac{5}{13},\ i_2=\frac{6}{13}$$

and

$$i_1-i_2=-\frac{1}{13}$$

(i) Potential difference between B and D,.

$$V_B+2(i_1-i_2)=V_D$$

$$\therefore \quad V_B-V_D=-2(i_1-i_2)=\frac{2}{13}\text{V}$$

(ii) $V_G=E_G-i_2r_G=3-\dfrac{6}{13}\times 3=\dfrac{21}{13}$ V

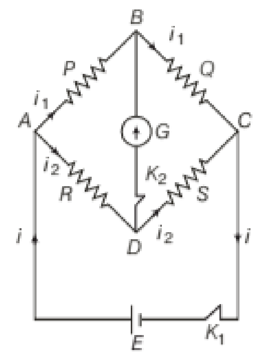
$$V_H=E_H+i_2r_H=1+\frac{6}{13}\times 1=\frac{19}{13}\text{ V}$$

WHEATSTONE BRIDGE

It is an arrangement of four resistances used to measure one of them in terms of the other three.

Consider four resistances P, Q, R and S are connected in the four arms of a quadrilateral. The galvanometer G and a tapping key K_2 are connected between points B and D. The cell of emf E and 1-way key K_1 are connected between points A and C as shown in the figure. Resistances P and Q are called ratio arms, resistance R is a variable resistance and S is unknown resistance.

The bridge is said to be balanced, when the galvanometer gives zero deflection. Thus, we have balance condition as



Wheatstone bridge

$$\boxed{\frac{P}{Q}=\frac{R}{S}}$$

Proof

In figure, four resistances P, Q, R and S are connected in the four arms of a parallelogram $ABCD$. Between B and D there is a sensitive galvanometer and a cell is connected between A and C. K_1 and K_2 are two keys. By pressing the key K_1, a current i is allowed to flow from the cell. At the point A, the current i is divided into two parts.

One part i_1 flows in the arm AB and the other part i_2 flows in the arm AD. The resistances P, Q, R and S are so adjusted that on pressing the key K_2 there is no deflection in the galvanometer G. That is, there is no current in the diagonal BD. Thus, the same current i_1 will flow in the arm BC as in the arm AB and the same i_2 will flow in the arm DC as in the arm AD.

Applying Kirchhoff 's second law for the closed loop $BADB$, we have

$$-i_1P+i_2R=0$$

$$Pi_1=Ri_2 \quad \text{...(i)}$$

Similarly, for the closed loop $CBDC$, we have

$$-i_1Q + i_2S = 0$$
$$Qi_1 = Si_2 \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{i_1P}{i_1Q} = \frac{i_2R}{i_2S} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

It is clear from this formula that if the ratio of the resistances P and Q and resistance R are known, then the unknown resistance S can be calculated. This is why, the arms AB and BC are called **ratio arms**, arm AD **known arm** and arm CD **unknown arm.**

When the bridge is balanced, then on interchanging the positions of the galvanometer and the cell there is no effect on the balance condition of the bridge. Hence, the arms BD and AC are called **conjugate arms** of the bridge. (In balanced state, no current flows in the galvanometer arm, hence while computing the equivalent resistance between A and C, the resistance connected between B and D may be neglected.) The sensitivity of the bridge depends upon the values of the resistance. The bridge is maximum sensitive, when all the four resistances are of the same order.

According to Maxwell, for greater sensitivity of the bridge, the galvanometer or the battery whichever has the higher resistance should be connected across the junctions of two highest and two lowest resistances.

Note The Wheatstone bridge is most sensitive, when the resistance of all the four arms of the bridge is of same order (or same), i.e. null point is obtained at the middle of bridge wire.

The advantage of null point method /zero deflection in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and internal resistance of a galvanometer.

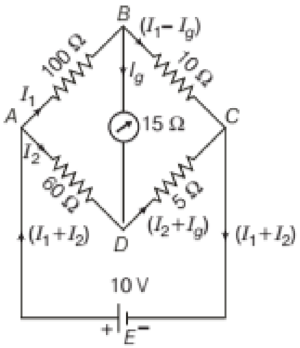
EXAMPLE |4| In a Wheatstone bridge circuit, $P = 7\,\Omega$, $Q = 8\,\Omega$, $R = 12\,\Omega$ and $S = 7\,\Omega$. Find the additional resistance to be used in series with S, so that the bridge is balanced.

Sol. Let the bridge be balanced when additional resistance x is put in series with S.

Then, $(S + x) = \frac{Q}{P}R$

or $$x = \frac{Q}{P}R - S = \frac{8}{7} \times 12 - 7 = 6.72\Omega$$

EXAMPLE |5| The Wheatstone bridge circuit have the resistances in various arms as shown in figure. Calculate the current through the galvanometer.



Sol. In the closed loop $ABDA$,

$$100\,I_1 + 15\,I_g - 60\,I_2 = 0$$
$$\Rightarrow \quad 20\,I_1 + 3\,I_g - 12\,I_2 = 0 \quad \text{...(i)}$$

In the closed loop $BCDB$,

$$10(I_1 - I_g) - 5(I_2 + I_g) - 15I_g = 0$$
$$\Rightarrow \quad 10I_1 - 30\,I_g - 5\,I_2 = 0$$
$$\Rightarrow \quad 2I_1 - 6\,I_g - I_2 = 0 \quad \text{...(ii)}$$

In the closed loop $ADCEA$,

$$60\,I_2 + 5(I_2 + I_g) = 10$$
$$\Rightarrow \quad 65\,I_2 + 5\,I_g = 10$$
$$\Rightarrow \quad 13I_2 + I_g = 2 \quad \text{...(iii)}$$

On solving Eqs. (i), (ii) and (iii), we get

$$I_g = 4.87 \text{ mA}$$

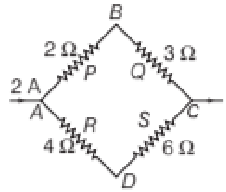
TOPIC PRACTICE 4

OBJECTIVE Type Questions

1. Kirchhoff's current law is consequence of conservation of

(a) energy (b) momentum
(c) charge (d) mass

2. If 2 A current is flowing in the shown circuit, then potential difference $(V_B - V_D)$ in balanced condition is



(a) 12 V (b) 6 V
(c) 4 V (d) zero

3. The Wheatstone bridge and its balance condition provide a practical method for determination of an
(a) known resistance
(b) unknown resistance
(c) Both (a) and (b)
(d) None of the above

VERY SHORT ANSWER Type Questions

4. State Kirchhoff's first law. **All India 2010**

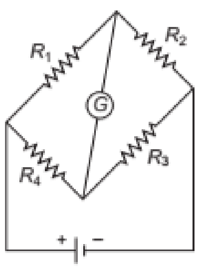
5. State Kirchhoff's second law.

6. When a Wheatstone bridge is most sensitive?

SHORT ANSWER Type Questions

7. Use Kirchhoff's rules to obtain the balance condition in a Wheatstone bridge. **Delhi 2012**

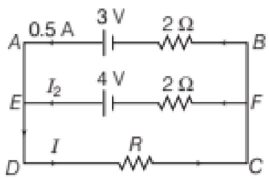
8. For the circuit diagram of a Wheatstone bridge shown in the figure, use Kirchhoff's laws to obtain its balance condition.



Delhi 2009

NUMERICAL PROBLEMS

9. Using Kirchhoff's rules in the given circuit, determine

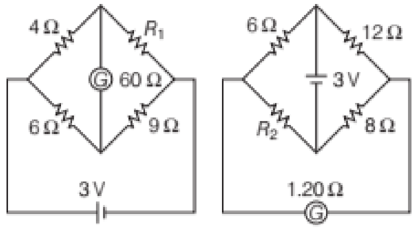


(i) the voltage drop across the unknown resistor R.
(ii) the current I in the arm EF. **All India 2011**

10. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of 1 Ω resistance. Use Kirchhoff's rules to determine

(i) the equivalent resistance of the network.
(ii) the total current in the network.
All India 2010

11. Figure shows two circuits each having a galvanometer and a battery of 3 V.
When the galvanometer in each arrangement do not show any deflection, obtain the ratio R_1/R_2. **All India 2013**



HINTS AND SOLUTIONS

1. (c) According to Kirchhoff's law, the algebraic sum of the currents is meeting at point in an electrical circuit is always zero, i.e. at any junction, the charge cannot be stored and cannot be loss. So, Kirchhoff's current law is consequence of conservation of charge.

2. (d) In Wheatstone bridge, $\frac{P}{Q} = \frac{R}{S}$

or $$\frac{2}{3} = \frac{4}{6} = \frac{2}{3}$$

i.e. in the balanced condition, $V_B - V_D = 0$

3. (b) In meter bridge balanced wheatstone bridge is used to determine unknown resistance.

4. Kirchhoff's first law states that the algebraic sum of currents at a junction in an electrical circuit is zero, i.e. $\Sigma I = 0$.

5. Kirchhoff's second law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

6. The Wheatstone bridge is most sensitive, when the resistance of all the four arms of the bridge are equal.

7. Refer to text on page 154.

8. Refer to text on page 154.
Put $P = R_1, Q = R_2, R = R_4$
and $S = R_3$.

9. (i) Applying Kirchhoff's second rule in the closed loop $ABFEA$,

$$V_B - 0.5 \times 2 + 3 = V_A$$
$$\Rightarrow \quad V_B - V_A = -2$$
$$V = V_A - V_B = +2\text{ V}$$

Potential drop across R is 2 V as R, EF and upper row are in parallel.

(ii) Applying Kirchhoff's first rule at E,

$$0.5 + I_2 = I$$

where, I is current through R.

Now, Kirchhoff's second rule in closed loop $FEABF$,

$$-2I_2 + 4 - 3 + 0.5 \times 2 = 0$$

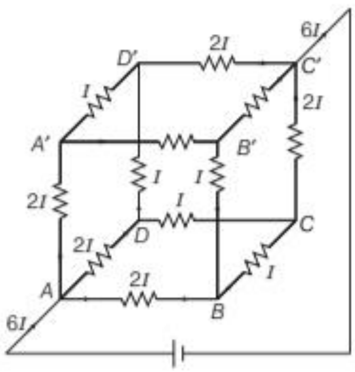
$$2I_2 - 2 = 0$$

or $\quad I_2 = 1\text{ A}$

The current in arm $EF = 1\text{ A}$

10. Let $6\,I$ current be drawn from the cell. Since, the paths AA', AD and AB are symmetrical, current through them is same.

As per Kirchhoff's junction rule, the current distribution is shown in the figure.



(i) Let the equivalent resistance across the combination be R.

$$\therefore \quad E = V_A - V_B$$

$$= (6I)\,R$$

$$\Rightarrow \quad 6IR = 10 \qquad [\because E = 10\text{ V}] \ldots\text{(i)}$$

(ii) Applying Kirchhoff 's second rule in loop $AA'B'C'A$,

$$-2I \times 1 - I \times 1 - 2I \times 1 + 10 = 0$$

$$\Rightarrow \quad 5I = 10$$

$$\Rightarrow \quad I = 2\text{ A}$$

Total current in the network

$$= 6I = 6 \times 2 = 12\text{ A}$$

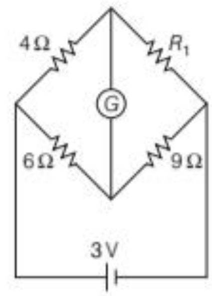
From Eq. (i), we get

$$6IR = 10$$

$$\Rightarrow \quad 6 \times 2 \times R = 10$$

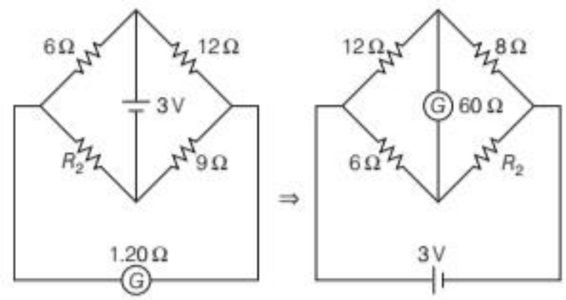
$$\Rightarrow \quad R = \frac{10}{12} = \frac{5}{6}\,\Omega$$

11.



For balanced Wheatstone bridge, there will be no deflection in the galvanometer.

$$\frac{4}{R_1} = \frac{6}{9} \Rightarrow R_1 = \frac{4 \times 9}{6} = 6\,\Omega$$



For the equivalent circuit, when the Wheatstone bridge is balanced, there will be no deflection in the galvanometer.

$$\therefore \quad \frac{12}{8} = \frac{6}{R_2}$$

$$\Rightarrow \quad R_2 = \frac{6 \times 8}{12} = 4\,\Omega$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{6}{4} = \frac{3}{2}$$

SUMMARY

- **Electric Current** It is defined as the rate of flow of electric charge through any cross-section of a conductor, i.e.
$$I = (dq / dt)$$
- The directed rate of flow of electric charge through any cross-section of a conductor is known as **electric current**.
$$I = \frac{q}{t} = \frac{ne}{t} \qquad [\because q = ne]$$
where, n = number of charged particles constitute the current
- **Current Density** It is the ratio of the current at a point in conductor to the area of cross-section of the conductor at that point, i.e. $J = (I / A)$.
- **Ohm's Law** At constant temperature, the potential difference V across the ends of a given metallic wire (conductor) in an circuit (electric) is directly proportional to the current flowing through it.

$$V \propto I$$

The variation of current w.r.t. applied potential difference is shown with the help of given graph.
$$V = IR$$
where, R = resistance of conductor.
- **Flow of Electric Charges in Metallic Conductors** In case of solid conductor, large number of free electrons causes the strong current in them.

  In the case of a liquid conductor, movement of positive and negative charged ions causes the electric current.
- **Resistance of a Conductor** Mathematically, it is the ratio of potential difference applied across the ends of conductor to the current flowing through it.
$$\Rightarrow \qquad R = \frac{V}{I}$$
SI unit is ohm (Ω).

  Resistance can also be written as, $R = \rho \frac{L}{A}$.

  where, L = length of the conductor,
  A = area of cross-section
  and ρ = constant, known as resistivity of the material.

  It depends upon nature of the material.
- **Effect of Temperature on Resistance**

  For metals, resistance increases with rise in temperature.

  For insulators and semiconductors, resistance decreases with rise in temperature.

  For alloys, temperature coefficient of resistance is small.
- **Temperature coefficient** of resistance is given by
$$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$
- **Drift Velocity** It is defined as the average velocity with which the free electrons move towards the positive end of a conductor under the influence of an external electric field applied.
$$\Rightarrow \qquad \mathbf{v}_d = \frac{e\mathbf{E}}{m} \tau$$
where, τ = relaxation time,
E = electric field,
m = mass and e = electron.
- **Relation between Drift Velocity and Electric Field**

  It is given by, $I = neAv_d$

  where, n = number density of free electrons,
  e = electronic charge,
  A = cross-sectional area
  and v_d = drift velocity of an electron.
- The ratio of drift velocity of electrons and the applied electric field is known as **mobility**.
$$\Rightarrow \qquad \mu = \frac{v_d}{E} = \frac{q\tau}{m}$$
$\therefore$ SI unit is $[m^2 s^{-1} V^{-1}]$.
- **Resistivity** It is the resistance of a unit length with unit area of cross-section of the material of the conductor.
- **Relationship between resistivity and relaxation time**
$$\rho = \frac{m}{ne^2 \tau}$$
where, τ = relaxation time.

  Specific resistance or resistivity (ρ) depends on the material of conductor, not on the length and cross-sectional area (A), i.e. geometry of conductor.
- **Effect of Temperature on Resistivity**

  For metals, resistivity increases with increase in temperature.

  For semiconductor, resistivity decreases with increase in temperature.

  For alloys, resistivity is very large but has a weak dependence on temperature.
- **Classification of Materials in terms of Conductivity**

  For insulators, electrical conductivity is very small or nil.

  For conductors, electrical conductivity is very high.

  For semiconductors, electrical conductivity lies in between that of insulators and conductors.

- **Conductance and Conductivity**

  Conductance is the reciprocal of resistance of conductor.

  Conductivity is the reciprocal of the resistivity of conductor.

- **Electrical Energy and Power** Electrical energy is the total work done in maintaining the electric current in the given circuit for a specified time.

  Electrical power is the rate of electrical energy supplied per unit time to maintain flow of electric current through conductor.

- **Internal Resistance and Electromotive Force of a Cell**

  EMF (E) It is the maximum potential difference between two terminals of circuit, when circuit is open.

  Internal Resistance (r) The resistance offered by the electrolyte of the cell, to the flow of current through it.

  Terminal Potential Difference (V) It is the maximum potential difference between two terminals of circuit, when the circuit is closed.

- **The relationship between *r*, *R*, *E* and *V*** is

$$r = R\left(\frac{E}{V} - 1\right) \quad \ldots\text{(i)}$$

where, $r =$ internal resistance, $R =$ external resistance,

$E =$ emf of cell, $V =$ terminal voltage of cell.

Also, $V = E - Ir = \left(\frac{E}{R+r}\right)R \quad \ldots\text{(ii)}$

- **Combination of Cells**

  In series grouping, current is given by, $I = (n\varepsilon / R + nr)$.

  In parallel grouping, current is given by, $I = (m\varepsilon / r + mR)$.

  In mixed grouping, current is given by, $I = \left(\frac{mn\varepsilon}{nr + mR}\right)$.

- **Kirchhoff's Laws**

  First Law (Junction Rule) The algebraic sum of the currents meeting at a point in an electrical circuit is always zero.

  Second Law (Loop Rule) The algebraic sum of changes in potential around any closed loop involving resistors and the cells in the loop is zero.

- **Wheatstone Bridge** It is an arrangement of four resistances used to measure one of them in terms of another three. The bridge is said to be balanced when the galvanometer shows zero deflection.

  The balance condition is $\frac{P}{Q} = \frac{R}{S}$.

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A potential difference V is applied to a copper wire of length l and diameter d. If V is doubled, then the drift velocity

(a) is doubled (b) is halved
(c) remains same (d) becomes zero

2. A potential difference of 100 V is applied to the ends of a copper wire one metre long. What is the average drift velocity of electrons?
(given, $\sigma = 5.81\times10^{7}\,\Omega^{-1}$ or $n_{Cu} = 8.5\times10^{28}\text{ m}^{-3}$)

(a) 0.43 ms^{-1} (b) 0.83 ms^{-1}
(c) 0.52 ms^{-1} (d) 0.95 ms^{-1}

3. Unit of specific resistance is

(a) ohm^{-1}-m^{-1} (b) ohm^{-1}-m
(c) ohm-m^{-1} (d) ohm-m

4. The length of 50 Ω resistance becomes twice by stretching. The new resistance is

(a) 25 Ω (b) 50 Ω (c) 100 Ω (d) 200 Ω

5. A metal rod of length 10 cm and a rectangular cross-section of $1\text{cm}\times\frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be

(a) maximum when the battery is connected across $1\text{ cm}\times\frac{1}{2}$ cm faces
(b) maximum when the battery is connected across $10\text{ cm}\times1$ cm faces
(c) maximum when the battery is connected across $10\text{ cm}\times\frac{1}{2}$ cm faces
(d) same irrespective of the three faces

6. The electric power consumed by a 220 V-100 W bulb, when operated at 110 V is **CBSE 2021 (Term-I)**

(a) 25 W (b) 30 W (c) 35 W (d) 45 W

7. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a **CBSE 2021 (Term-I)**

(a) low resistance in parallel
(b) low resistance in series
(c) high resistance in parallel
(d) high resistance in series

8. Kirchhoff's first rule, $\Sigma I = 0$ and second rule, $\Sigma IR = \Sigma E$ (where the symbols have their usual meanings) are respectively, based on **CBSE 2021 (Term-I)**

(a) conservation of momentum and conservation of charge
(b) conservation of energy and conservation of charge
(c) conservation of charge and conservation of momentum
(d) conservation of charge and conservation of energy

9. Which of the following has negative temperature coefficient of resistivity? **CBSE 2021 (Term-I)**

(a) Metal
(b) Metal and semiconductor
(c) Semiconductor
(d) Metal and alloy

10. If the potential difference V applied across a conductor is increased to $2V$ with its temperature kept constant, then the drift velocity of the free electrons in a conductor will **CBSE SQP (Term-I)**

(a) remain the same
(b) become half of its previous value
(c) be double of its initial value
(d) become zero

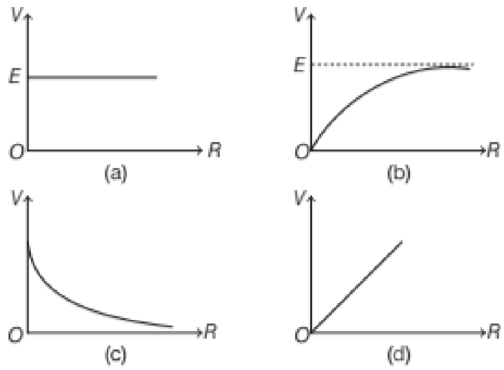
11. A constant voltage is applied between the two ends of a uniform metallic wire, heat H is developed in it. If another wire of the same material, double the radius and twice the length as compared to original wire is used, then the heat developed in it will be **CBSE SQP (Term-I)**

(a) $H/2$ (b) H
(c) $2H$ (d) $4H$

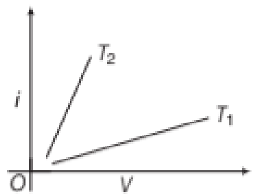
12. In a DC circuit, the direction of current inside the battery and outside the battery, respectively are **CBSE SQP (Term-I)**

(a) positive to negative terminal and negative to positive terminal
(b) positive to negative terminal and positive to negative terminal
(c) negative to positive terminal and positive to negative terminal
(d) negative to positive terminal and negative to positive terminal

13. A cell of emf (E) and internal resistance r is connected across a variable external resistance R. The graph of terminal potential difference V as a function of R is **CBSE 2020**



14. The current i and voltage V graph for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure. It is concluded that



(a) $T_1 > T_2$ (b) $T_1 < T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

15. The electromotive force of cell is 5V and its internal resistance is 2 Ω. This cell is connected to external resistance. If the current in the circuit is 0.4 A, then voltage of poles of cell is

(a) 5 V (b) 5.8 V (c) 4.6 V (d) 4.2 V

ASSERTION AND REASON

Directions (Q. Nos. 16-21) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

16. **Assertion** The average time of collisions τ, decreases with increasing temperature.
Reason At increased in temperature, average speed of the electrons, which act as the carriers of current, increases, resulting in more frequent collisions.

17. **Assertion** Charge carriers do not move with acceleration, with a steady drift velocity.
Reason Charge carriers under go collisions with ions and atoms during transit.

18. **Assertion** If we bend an insulated conducting wire, the resistance of the wire increases.

Reason The drift velocity of electron in bended wire remains same.

19. **Assertion** The drift velocity of electrons in a metallic wire decreases when temperature of the wire is increases.

Reason On increasing temperature, conductivity of metallic wire decreases.

20. **Assertion** Manganin and constantan are widely used in standard resistors.
Reason Manganin and constantan resistances values would change very little with temperatures.

21. **Assertion** Higher the range, lower is the resistance of an ammeter.

Reason To increase the range of an ammeter, additional shunt is added in series to it.

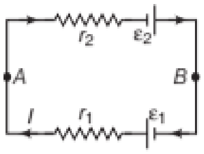
CBSE 2021 (Term-I)

CASE BASED QUESTIONS

Directions (Q.Nos. 22-23) *This question is case study based question. Attempt any 4 sub-parts from this question. Each question carries 1 mark.*

22. **Potential Difference**

The potential difference (V) across a source in a circuit is not equal to its emf (ε). This is due to the reason that every source of electric energy has some internal resistance (r). Further, ε, V and r are related to each other as $V = \varepsilon - Ir$. A single battery shown in figure, consists of two cells of emf's ε_1 and ε_2 and internal resistances r_1 and r_2, respectively in series.



(i) The current in the internal circuit is

(a) zero (b) $\frac{\varepsilon_2 - \varepsilon_1}{r_1 + r_2}$

(c) $\frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}$ (d) $\frac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2}$

(ii) The equivalent emf of the battery is

(a) $(\varepsilon_1 + \varepsilon_2)$ (b) $(\varepsilon_1 - \varepsilon_2)$

(c) $(\varepsilon_2 - \varepsilon_1)$ (d) $\frac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)}{r_1 + r_2}$

(iii) For the terminal A to be positive

(a) $\varepsilon_1 > \varepsilon_2$ (b) $\varepsilon_2 > \varepsilon_1$

(c) $\varepsilon_1 r_1 = \varepsilon_2 r_2$ (d) $\varepsilon_1 r_2 > \varepsilon_2 r_1$

(iv) The internal resistance of the battery is

(a) $(r_1 + r_2)$ (b) $\frac{(r_1 + r_2)}{r_1 r_2}$

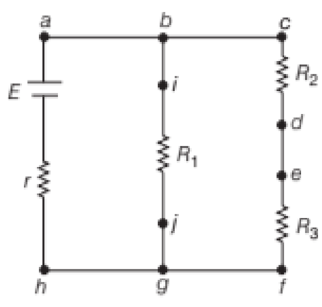
(c) $\frac{r_1 r_2}{(r_1 + r_2)}$ (d) $\frac{r_2}{(r_1 + r_2)}$

(v) The algebraic sum of changes in potential around any closed loop involving resistor and cells in the loop is

(a) more than zero (b) less than zero

(c) zero (d) constant

23. An experiment was set-up with the circuit diagram shown in figure. Given that, $R_1 = 10\,\Omega$, $R_2 = R_3 = 5\Omega$, $r = 0\,\Omega$ and $E = 5\,V$ **CBSE 2021 (Term-I)**



(i) The points with the same potential are

(a) b, c, d (b) f, h, j (c) d, e, f (d) a, b, j

(ii) The current through branch bg is

(a) 1 A (b) $\frac{1}{3}$ A (c) $\frac{1}{2}$ A (d) $\frac{2}{3}$ A

(iii) The power dissipated in R_1 is

(a) 2 W (b) 2.5 W

(c) 3 W (d) 4.5 W

(iv) The potential difference across R_3 is

(a) 1.5 V (b) 2 V (c) 2.5 V (d) 3 V

FILL IN THE BLANK

24. A copper wire of non-uniform area of cross-section is connected to a DC battery. The physical quantity which remains constant along the wire is ....

VERY SHORT ANSWER Type Questions

25. What is the significance of direction of electric current?

26. Describe how the resistivity of the conductor depends upon

(i) number density (n) of free electrons and

(ii) relaxation time (τ).

27. Two conducting wires A and B of the same length but of different materials are joined in series across a battery. If the number density of electrons in A is twice than that in B, find the ratio of drift velocities of electrons in two wires.

28. How does the mobility of electrons in a conductor change, if the potential difference applied across the conductor is doubled, keeping the length and temperature of the conductor constant? **CBSE 2019**

29. When a potential difference is applied across the ends of a conductor, how is the drift velocity of the electrons related to the relaxation time? **CBSE 2019**

30. How is the drift velocity in a conductor affected with the rise in temperature? **CBSE 2019**

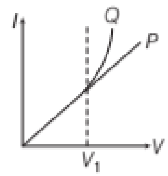
31. Show variation of resistivity of copper as a function of temperature in graph.

32. On what basic conservation laws, are Kirchhoff's laws based?

33. Define the conductivity of a conductor. Write its SI unit. **All India 2017 C**

SHORT ANSWER Type Questions

34. Figure below shows a plot of current *versus* voltage for two different materials P and Q. Which of the two materials satisfies Ohm's law? Explain.



35. Derive the expression for the resistivity of a good conductor in terms of the relaxation time of electrons.

36. Write the expression for the resistivity of a metallic conductor showing its variation over a limited range of temperatures.

37. Car batteries are often rated in unit ampere hours. Does this unit designate the amount of current, energy, power or charge that can be drawn from the battery? Explain.

38. Two bulbs are rated (P_1, V) and (P_2, V). If they are connected (i) in series and (ii) in parallel across a supply V, find the power dissipated in the two combinations in terms of P_1 and P_2. **CBSE 2020**

39. A wire of length L_0 has a resistance R_0. It is gradually stretched till its length becomes $2L_0$. **CBSE 2020**

(a) Plot a graph showing variation of its resistance R with its length L during stretching.

(b) What will be its resistance when its length becomes $2\,L_0$?

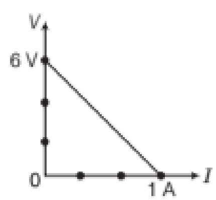
40. A wire of length L_0 has a resistance R_0. It is gradually stretched till its length becomes 1.5 L_0.

(a) Plot a graph showing variation of its resistance R with its length l during stretching.

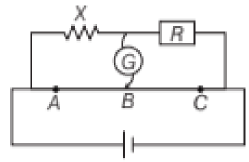
(b) What will be its resistance when its length becomes 1.5 L_0?

41. Is there some net field inside the cell, when the circuit is closed and a steady current passes through? Explain.

42. The plot of the variation of potential difference across a combination of three identical cells in series *versus* current is as shown in the figure. What is the emf of each cell?



43. R_1, R_2 and R_3 are three different values of resistor R. Such that $R_1 > R_2 > R_3$
A, B and C are the null points obtained corresponding to R_1, R_2 and R_3, respectively. For which resistor, the value of X will be most accurate and why?

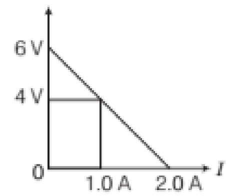


44. The figure shows a plot of terminal voltage V *versus* the current I of a given cell.

Calculate from the graph

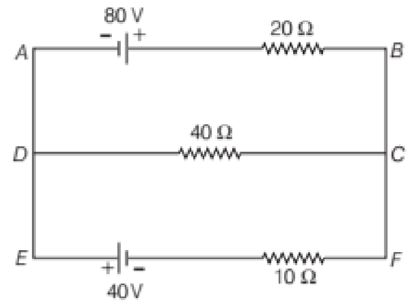
(i) emf of the cell.

(ii) internal resistance of the cell. **All India 2017C**



LONG ANSWER Type I Questions

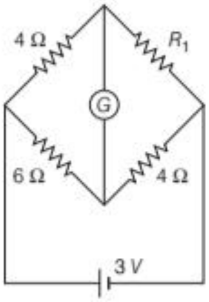
45. Using Kirchhoff's rules, calculate the current through the 40 Ω and 20 Ω resistors in the following circuit. **CBSE 2019**

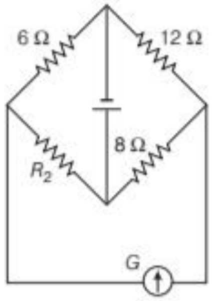


46. Show on a plot, variation of resistivity of (i) a conductor and (ii) a typical semiconductor as a function of temperature.
Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature. **CBSE 2019**

47. With the help of a suitable diagram, explain in brief about the sensitivity of Wheatstone bridge?

48. Define the term current sensitivity of a galvanometer. In the circuits shown in the figures, the galvanometer shows no deflection in each case. Find the ratio of R_1 and R_2.





All India 2017C

LONG ANSWER Type II Questions

49. A variable resistor R is connected across a cell of emf E and internal resistance r.

CBSE SQP (Term-I)

(a) Draw the circuit diagram.

(b) Plot the graph showing variation of potential drop across R as function of R.

(c) At what value of R, current in circuit will be maximum?

50. A storage battery is of emf 8V and internal resistance 0.5 Ω is being charged by DC supply of 120 V using a resistor of 15.5 Ω.

CBSE SQP (Term-I)

(a) Draw the circuit diagram.

(b) Calculate the potential difference across the battery.

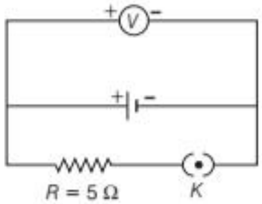
(c) What is the purpose of having series resistance in this circuit?

NUMERICAL PROBLEMS

51. At 20°C, the carbon resistor in an electric circuit connected to a 5 V battery has a resistance of 200 Ω. What is the current in the circuit when the temperature of the carbon rises to 80° C?

52. A semiconductor has electron concentration $0.45 \times 10^{12}\ m^3$ and hole concentration $5 \times 10^{20}\ m^3$. Find its conductivity. Given, electron mobility = 0.135 m^2/Vs and hole mobility = 0.048 m^2/Vs, $e = 1.6 \times 10^{-19}$C.

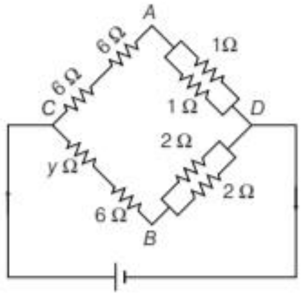
53. Write any two factors on which internal resistance of a cell depends. The reading on a high resistance voltmeter when a cell is connected across it is 2.2 V.



When the terminals of the cell are also connected to a resistance of 5 Ω as shown in the circuit, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.

54. The emf of a battery is 2 V and its internal resistance is 2 Ω. Its potential difference is measured by a voltmeter of resistance 998 Ω. Calculate the percentage error in the reading of emf shown by the voltmeter.

55. For what value of unknown resistance y, the potential difference between A and B is zero in the arrangement as shown in figure given below?



56. The resistance of a potentiometer wire of length 10 m is 20 Ω. A resistance box and a 2V accumulator are connected in series with it. What resistance should be introduced in the box to have a potential drop of 1 μV/mm of the potentiometer wire?

ANSWERS

1. (a)	2. (a)	3. (d)	4. (d)	5. (a)
6. (a)	7. (a)	8. (d)	9. (c)	10. (c)
11. (c)	12. (c)	13. (b)	14. (a)	15. (d)

16. (*a*) With increase in temperature, average speed of the electrons, (which acts as the carriers of current) increases resulting in more frequent collisions. The average time of collisions τ, thus decreases with temperature.

17. (*a*) Charge carriers do not move with acceleration but with a steady drift velocity. This is because of the collisions with ions and atoms during transit.

18. (*d*) Bending will not increase the resistance of the conducting wire. Also drift velocity of electron is independent of bending of conductor.

19. (*b*) Increasing the temperature of a conductor, the kinetic energy of free electrons increases.
On account of this, they collide more frequently with each other (and with the ions of the conductor) and consequently their drift velocity decreases.

20. (*a*) Manganin and constantan have very low temperature coefficient resistance.

21. (a)

22. (i) (*c*) Net current, $I = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}$

(ii) (*d*) Equivalent emf of battery

$$\varepsilon = V_A - V_B = \varepsilon_1 - Ir_1$$
$$= \varepsilon_1 - \left(\frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}\right) r_1 = \frac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)}{(r_1 + r_2)}$$

(iii) (*d*) Terminal A is positive, if $V_A > V_B$ or $V_A - V_B > 0$

or $(\varepsilon_1 r_2 - \varepsilon_2 r_1) > 0$

or $\varepsilon_1 r_2 > \varepsilon_2 r_1$

(iv) (*a*) Since, r_1 and r_2 are in series, so resultant resistance is

$$r = r_1 + r_2$$

(v) (*c*) **Loop Rule** The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. This rule is also obvious, since, electric potential is dependent on the location of the point. Thus, starting with any point, if we come back to the same point, the total change must be zero. In a closed loop, we do come back to the starting point and hence the rule. $\Sigma Ir = \Sigma V$

23. (i) (*b*), (ii) (*c*), (iii) (*b*), (iv) (*c*)

24. Current

25. Electric current is caused by the flow of electrons in a conductor. But the direction of electric current is taken as the opposite direction of movement of electrons.

26. Resistivity of a conductor is given by $\rho = \dfrac{m}{ne^2\tau}$

(i) Resistivity $\rho \propto \dfrac{1}{n}$, where n is the number density of free electrons.

(ii) Resistivity $\rho \propto \dfrac{1}{\tau}$, where τ is the relaxation time.

27. As the wires A and B are joined in series, the current through them is same.

$$I_A = I_B$$

$$(neAv_d)_A = (neAv_d)_B \quad [\text{as } I = ne\, Av_d]$$

$$\Rightarrow \quad n_A v_{d_A} = n_B v_{d_B}$$

$$\frac{v_{d_A}}{v_{d_B}} = \frac{n_B}{n_A} = \frac{1}{2}$$

28. The mobility of electrons in a conductor is given by

$$\mu = \frac{e\tau}{m}$$

where, e = charge on electron, m = mass of electrons and τ = relaxation time.

Also, $\tau \propto T$.

But here temperature (T) is kept constant. As mobility is independent of potential difference, so there is no change in it.

29. Average drift velocity, $v_d = \dfrac{eE}{m}\tau$

where, e = charge on electron,
m = mass of electron,
E = electric potential or field across conductor
and τ = relaxation time.

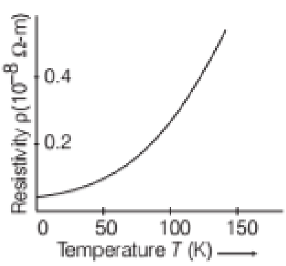
30. The average drift velocity, $v_d = \dfrac{eE}{m}\tau$

where, τ = relaxation time.
The relaxation time is directly proportional to the temperature of conductor i.e.

$$\tau \propto T$$
$$\therefore \quad v_d \propto T$$

So, the drift velocity increases with rise in temperature.

31. Graph of resistivity of copper as a function of temperature is given below (resistivity of metals increases with increase in temperature).



32. Kirchhoff's current law is based on law of conservation of charge and Kirchhoff's voltage law is based on law of conservation of energy.

33. Refer to text on page 130.

34. The plot of V *versus* I is a straight line for materials that obey Ohm's law. So, from the figure, material P obeys Ohm's law.

35. Refer to text on pages 128 and 129.

36. Refer to text on pages 129 and 130.

37. Ampere hours is the unit of charge as ampere is the unit of current and hours is the unit of time.
Charge = Current × Time.

38. The resistance P_1 is $R_1 = \dfrac{V^2}{P_1}$

and that P_2 is $R_2 = \dfrac{V^2}{P_2}$

(i) In series, $R = R_1 + R_2$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{V}{R_1 + R_2}$$

and $$P = I^2 (R_1 + R_2)$$

$$= \frac{V^2}{(R_1 + R_2)^2}(R_1 + R_2)$$

$$= \left(\frac{1}{\frac{R_1}{V^2} + \frac{R_2}{V^2}}\right) = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2}} = \frac{P_1 P_2}{P_1 + P_2}$$

(ii) In parallel, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \Rightarrow \dfrac{V^2}{R} = \dfrac{V^2}{R_1} + \dfrac{V^2}{R_2}$

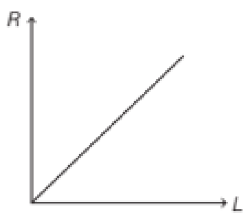
$$\boxed{P = P_1 + P_2}$$

39. (a) Initially, the resistance R_0 of a wire of length L_0 is given by

$$R_0 = \rho \frac{L_0}{A_0} \quad \text{...(i)}$$

$$\Rightarrow \quad R_0 \propto L_0$$

The variation of resistance R with length L during stretching is shown as



(b) When a wire is stretched to $2L_0$, the area of wire becomes $\dfrac{A_0}{2}$. So, the new resistance will be

$$R = \frac{\rho 2L_0}{A_0/2} = 4\rho \frac{L_0}{A_0} = 4R_0 \quad [\because \text{using Eq. (i)}]$$

$\therefore$ The resistance of wire becomes 4 times of previous.

40. (a) Refer to Q. 39 on page 162.

(b) 2.25 R_0; Refer to Q. 39 on page 162.

41. Refer to text on page 140.

42. When three identical cells are connected in series, the equivalent emf is given by

$$E_{eq} = E_1 + E_2 + E_3 = 3E$$

From the graph, $3E = 6\text{V}$

$$\Rightarrow \quad E = \frac{6}{3} = 2\text{V}$$

$\therefore$ Emf of each cell = 3V

43. The figure given is a potentiometer . The sensitivity of potentiometer can be increased by reducing the current in the circuit. This can be done by increasing the value of R. So, the value of X will be most accurate for R_1.

44. $V = E - ir$

(i) When $i = 0$, then $V = E$.

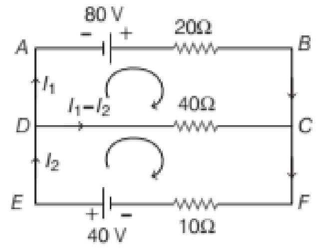
When $i = 0$, then $V = 6\text{V}$ (from the graph)

$\therefore$ emf of the cell $(E) = 6\text{V}$

(ii) When $i = 2\text{A}$, then $V = 0$ (from the graph)

$\therefore E = ir \Rightarrow r = \dfrac{E}{i} = \dfrac{6}{2} = 3\Omega$

45. Taking loops clockwise as shown in figure.



Using KVL in $ABCDA$,

$$-80 + 20I_1 + 40(I_1 - I_2) = 0$$

$$\Rightarrow \quad 3I_1 - 2I_2 = 4 \quad \text{...(i)}$$

Using KVL in $DCFED$,

$$-40(I_1 - I_2) + 10I_2 - 40 = 0$$

$$\Rightarrow -4I_1 + 5I_2 = 4 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$I_1 = 4$ A

and $I_2 = 4$ A

Thus, $I_{40} = I_1 - I_2 = 0$ A

$I_{20} = I_1 = 4$ A

46. (i) and (ii) Refer to text on pages 129 and 130. (Temperature Dependence of Resistivity)

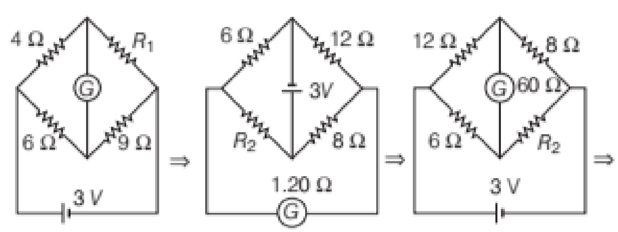
Refer to text on pages 128 and 129. (Resistivity of Various Materials)

47. Refer to text on pages 153 and 154.

48. Current sensitivity of a galvanometer is defined as the deflection per unit current.

For balanced Wheatstone bridge, there will be no deflection in the galvanometer.

$$\Rightarrow \quad \frac{4}{R_1} = \frac{6}{4} \Rightarrow R_1 = \frac{4 \times 4}{6} = \frac{8}{3}\,\Omega$$



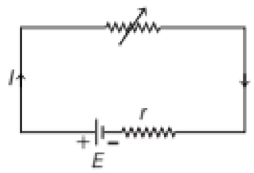
For the equivalent circuit, when the wheatstone bridge is balanced, there will be no deflection in the galvanometer.

$$\therefore \quad \frac{12}{8} = \frac{6}{R_2}$$

$$\Rightarrow \quad R_2 = \frac{6 \times 8}{12} = 4\Omega$$

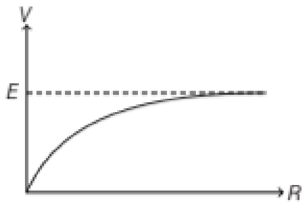
$$\therefore \quad \frac{R_1}{R_2} = \frac{8/3}{4} = \frac{2}{3}$$

49. (a) Circuit diagram



(b) Variation of potential drop across R as function of R is

$$\left[\because V = E\left(1 - \frac{1}{1 + r/R}\right)\right]$$



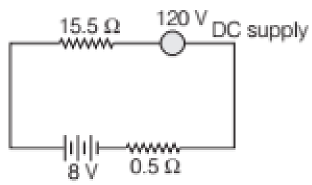
(c) The maximum value of current is obtained, when the resistance of external resistance R is zero,

i.e. $\quad I_{max} = \frac{E}{r}$

50. Given, $E = 8\text{ V}$, $\quad r = 0.5\ \Omega$,

$V = 120\text{ V}$ and $R = 15.5\ \Omega$

(a)



(b) Potential across battery in charging

$$V = E + Ir$$

Effective voltage, $V' = V - E = 120 - 8 = 112\text{ V}$

The current in circuit, $I = \frac{12}{15.5 + 0.5} = 7\text{ A}$

$$\Rightarrow V = 8 + 7 \times 0.5$$

$$= 8 + 3.5 = 11.5\text{ V}$$

(c) Series resistor in the charging circuit limits the current drawn from the external source.

51. Refer to text on page 129. [**Ans.** 26 mA]

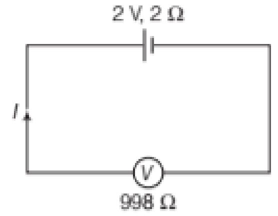
52. Refer to text on page 130. [**Ans.** 3.84 Sm^{-1}]

53. Refer to Example 1 on page 141. $\left[\textbf{Ans. } \frac{1}{9}\Omega\right]$

54. Given, $E = 2\text{V}$

$$r = 2\Omega$$

From the diagram,



$$I = \frac{2}{2 + 998} = \frac{2}{1000}\text{A}$$

$$V = E - Ir = 2 - \frac{2}{1000} \times 2$$

$$= 2 - 0.004 = 1.996\text{ V}$$

$$\therefore \%\text{ error} = \frac{0.004}{2} \times 100 = 0.2\%$$

55. As, $V_A - V_B = 0$

Thus, it is a balanced Wheatstone bridge. [**Ans.** 18 Ω]

56. Apply the balancing condition of a potentiometer

Given, $\quad E = 2\text{ volt}$

$V = 0.01\text{ volt}$

$R = 2\ \Omega/\text{m}$

Let r is resistance introduced in the box

$$\therefore \quad r = \left(\frac{E}{V} - 1\right)R$$

$$= \left(\frac{2}{0.01} - 1\right) \times 2$$

or $\quad r = 398\ \Omega$ [**Ans.** 398 Ω]